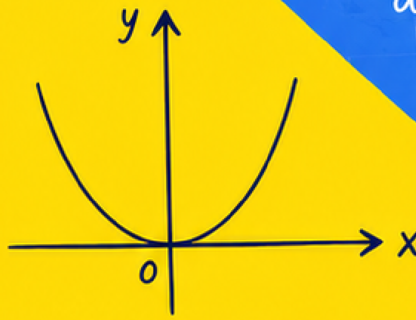
$$y = ax^2 + bx + c$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

ALGEBRA 2

FORMULA REVIEW

KEY FORMULAS. **CLEAR CONCEPTS.** CONFIDENT SOLUTIONS.

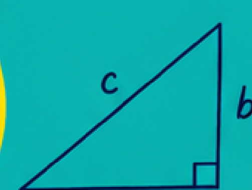
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$x^2 + y^2 = r^2$$

$$ax^2 + bx + c = a(x - r_1)(x - r_2)$$

$$\begin{cases} ax + by = c \\ dx + ey = f \end{cases}$$

COMPLETE FORMULAS | EASY TO UNDERSTAND | EXAM READY | YOUR REFERENCE. YOUR SUCCESS.



$$c^2 = a^2 + b^2$$

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COMPLETE FORMULA REVIEW & TUTORING GUIDE

ALGEBRA 2

Every Major Formula, Clearly Explained

Functions, equations, graphs, sequences, trig, conics, matrices, and data tools students use across Algebra 2.

Formula Snapshot

QUADRATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

all roots of $ax^2 + bx + c = 0$

LOGS

$$\log_b(MN) = \log_b M + \log_b N$$

$b > 0, \ b \neq 1, \ M, N > 0$

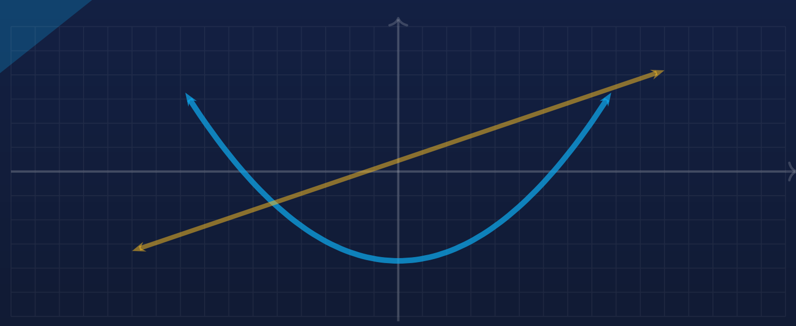
SEQUENCES

$$a_n = a_1 r^{n-1}$$

geometric pattern

Advanced Functions | Plain-English Tutor Notes | Worked Examples

Functions • Quadratics • Polynomials • Rational Expressions • Logs • Conics • Trigonometry • Statistics



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Welcome to the Algebra 2 Formula Review

Algebra 2 stretches Algebra 1 into a bigger toolkit: functions can be transformed, polynomials can have many roots, equations can create restrictions, and graphs can model growth, cycles, conics, and data. This guide is built to keep the key formulas in one place while still explaining what each one is for.

USE FIRST
Formula
Read the rule before starting practice.

WATCH FOR
Restrictions
Denominators, square roots, logs, and units matter.

THEN TRY
Example
Check one worked example before doing ten problems.

FINISH WITH
Practice
Scan the hub when a topic needs another pass.

HOW TO STUDY

Use this as a loop: read the formula, check the restriction, study the example, then practice the exact topic. If an answer comes from a squared equation, a rational equation, or a logarithm, plug it back into the original problem before trusting it.

How to read every section

Each topic uses the same color-coded blocks, so you always know what you are looking at.

Formula table	Tutor's Note	Example	Hint	Visual
the rules, with restrictions	plain-English meaning	one worked problem	common watch-outs	see the idea as a picture

The study loop

Run this cycle on each topic until it feels automatic — understanding first, then speed.

1. Read the formula & its restriction →
2. Study one worked example →
3. Practice that exact skill →
4. Check it in the original problem

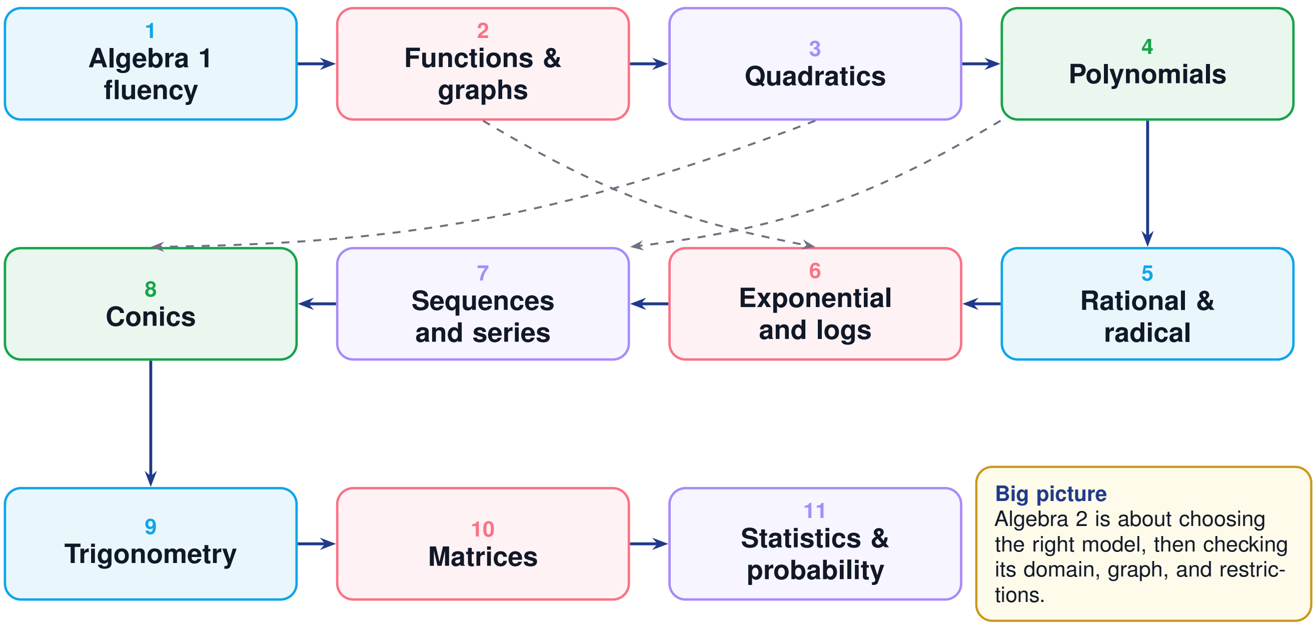
repeat on the next topic

What's Inside

Each section pairs the **formula** with a plain-English **Tutor's Note**, a worked example, and quick watch-outs.

Algebra 2 Roadmap

Algebra 2 connects families of functions. Linear and quadratic ideas become polynomial, rational, exponential, logarithmic, trigonometric, and statistical models.



ROADMAP

When a topic feels disconnected, ask: what function family is this, what does its graph look like, and what inputs are allowed?

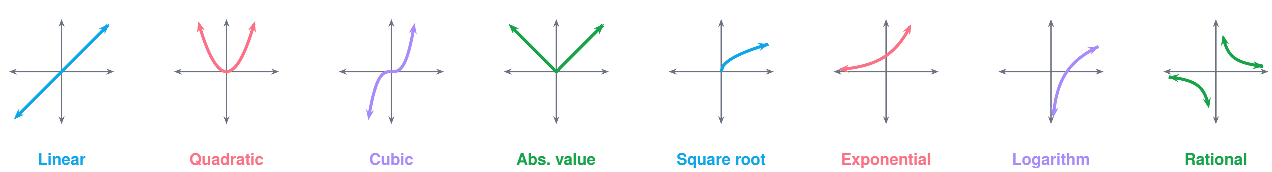
1. NAME THE FAMILY
Linear, quadratic, polynomial, rational, exponential, log, trig, or data?

2. CHOOSE THE TOOL
Use the form that reveals the target: zeros, vertex, rate, asymptote, or period.

3. CHECK THE ANSWER
Plug back into the original and check restrictions before moving on.

The function families at a glance

Most of Algebra 2 is recognizing which of these shapes a rule makes. Match the equation to the picture first, then reach for the matching formulas.



1 Functions, Transformations & Inverses

Core function formulas

Function notation $y = f(x)$ means the output when the input is x.

Average rate of change $\dfrac{f(b) - f(a)}{b - a}$, where $b \neq a$.

Composition $(f \circ g)(x) = f(g(x))$. Use the domain of g and any added restrictions from f.

Inverse functions $f^{-1}(x)$ reverses $f(x)$, and $f(f^{-1}(x)) = x$ on the allowed domain.

Transformation model $g(x) = a\,f(b(x - h)) + k$. Vertical shift k, horizontal shift h, vertical scale $|a|$, horizontal scale $\frac{1}{|b|}$.

Tutor's Note

Algebra 2 functions are machines you can move, stretch, combine, and reverse. Always track domain: denominators cannot be zero, even roots need nonnegative radicands, and logs need positive arguments.

Example

If $f(x) = x^2$ and $g(x) = 2f(x - 3) - 5$, then $g(x) = 2(x - 3)^2 - 5$. The graph shifts right 3, stretches vertically by 2, and shifts down 5.

INVERSE

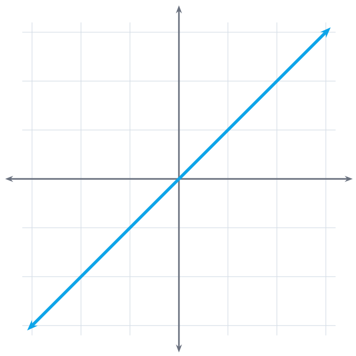
To find an inverse, write $y = f(x)$, swap x and y, solve for y, and then write $f^{-1}(x)$. If the original function is not one-to-one, restrict the domain before taking an inverse.

Before you move on A function rule, a line, and a system all answer the same question in different ways: **what output matches this input?** Check the input restrictions first, then use slope, substitution, or graph intersections to compare outputs.

2 Parent Functions & Transformations

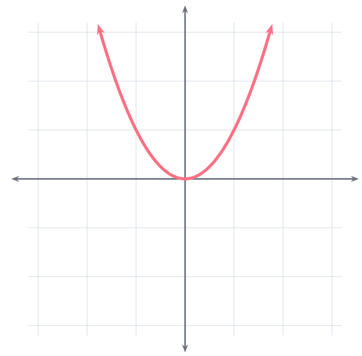
Every Algebra 2 graph is a transformed version of one of these parent shapes. Learn the parents — their graphs, domains, and ranges — then apply shifts, reflections, and stretches. Each curve continues past the arrows.

Linear $y = x$



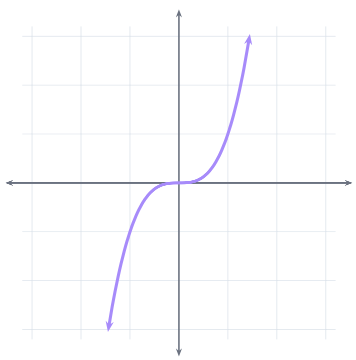
Domain $\mathbb{R}$ • Range $\mathbb{R}$

Quadratic $y = x^2$



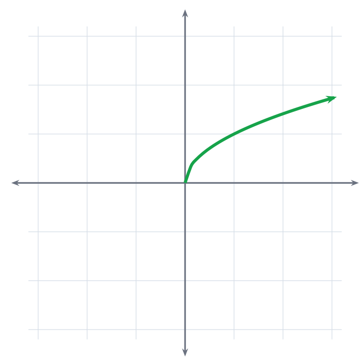
Domain $\mathbb{R}$ • Range $y \geq 0$

Cubic $y = x^3$



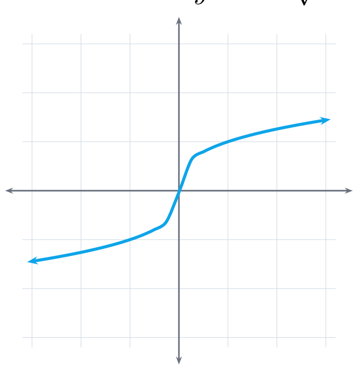
Domain $\mathbb{R}$ • Range $\mathbb{R}$

Square root $y = \sqrt{x}$



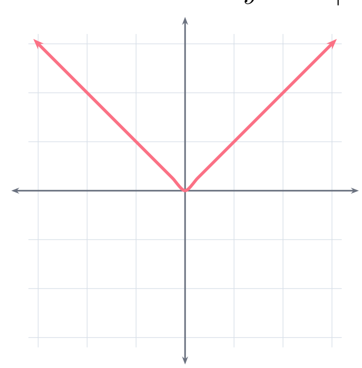
Domain $x \geq 0$ • Range $y \geq 0$

Cube root $y = \sqrt[3]{x}$



Domain $\mathbb{R}$ • Range $\mathbb{R}$

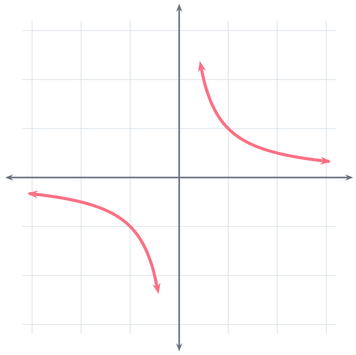
Absolute value $y = |x|$



Domain $\mathbb{R}$ • Range $y \geq 0$

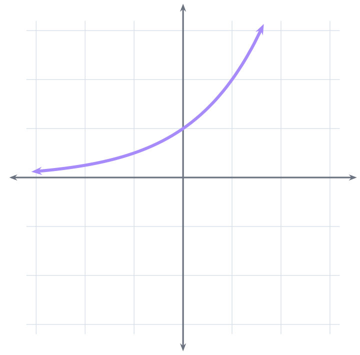
More parent functions

Reciprocal $y = \frac{1}{x}$



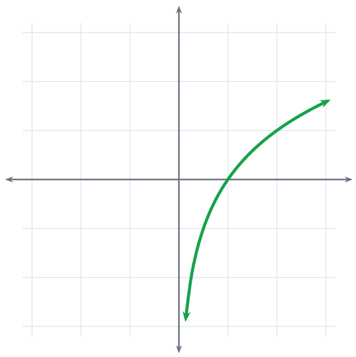
Domain $x \neq 0$ • Range $y \neq 0$

Exponential $y = b^x$



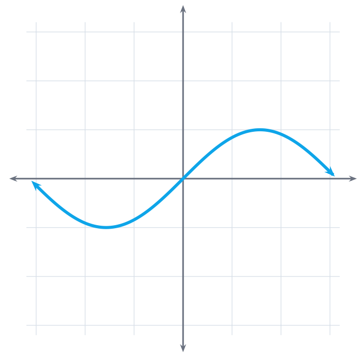
Domain $\mathbb{R}$ • Range $y > 0$

Logarithm $y = \log_b x$



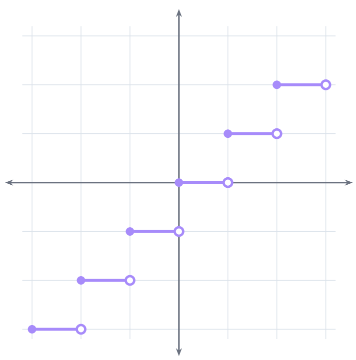
Domain $x > 0$ • Range $\mathbb{R}$

Sine $y = \sin x$



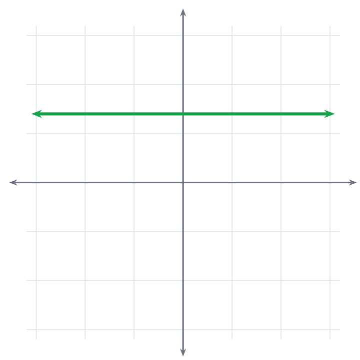
Domain $\mathbb{R}$ • Range $[-1, 1]$

Greatest integer $y = \lfloor x \rfloor$



Domain $\mathbb{R}$ • Range integers

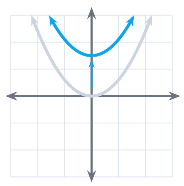
Constant $y = b$



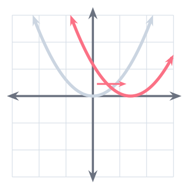
Domain $\mathbb{R}$ • Range $\{b\}$

Transformations of $y = x^2$

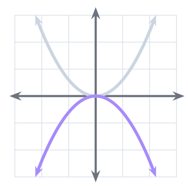
The gray curve is the parent $y = x^2$. Each colored curve shows one change from $g(x) = a\,f(x-h) + k$.



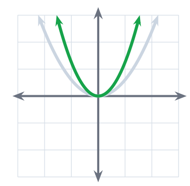
$+k$: **up** 1.5



$x - h$: **right** 1.4



$a < 0$: **reflect**



$a > 1$: **stretch**

Transformation rules for $g(x) = a\,f\big(b(x-h)\big) + k$

Vertical shift	$+k$ moves the graph up k; $-k$ moves it down.
Horizontal shift	$(x-h)$ moves the graph right h; $(x+h)$ moves it left.
Vertical stretch/shrink	$\lvert a\rvert > 1$ stretches; $0 < \lvert a\rvert < 1$ shrinks toward the x-axis.
Horizontal stretch/shrink	factor $\frac{1}{\lvert b\rvert}$; $\lvert b\rvert > 1$ shrinks, $0 < \lvert b\rvert < 1$ stretches.
Reflections	$a < 0$ reflects over the x-axis; $b < 0$ reflects over the y-axis.
Order of operations	stretch/reflect first, then shift; horizontal changes work in reverse.

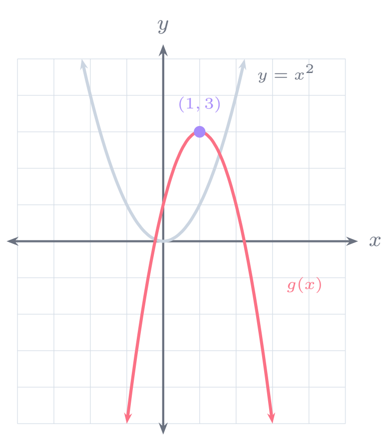
Tutor's Note

Read a transformed rule from the inside out. Inside the function, changes affect x and behave "backwards" (right/left, horizontal scaling). Outside the function, changes affect y and behave as written (up/down, vertical scaling, reflection).

DOMAIN & RANGE

Transformations move domain and range too. A vertical shift changes the range; a horizontal shift changes the domain; reflections can swap which way an inequality points.

Worked example: build $g(x) = -2(x-1)^2 + 3$ from the parent $y = x^2$



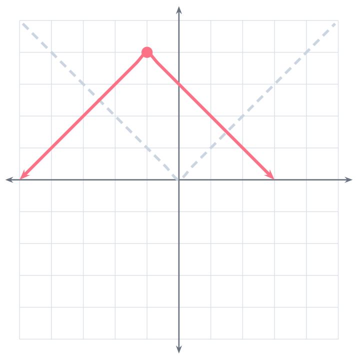
Read $g(x) = -2(x-1)^2 + 3$ as four moves on the parent $y = x^2$:

1. $x - 1$ inside: shift **right** 1.
2. $\times 2$: **stretch** vertically (narrower).
3. leading $-$: **reflect** over the x-axis (opens down).
4. $+3$ outside: shift **up** 3, so the vertex lands at $(1, 3)$.

Transforming every parent function

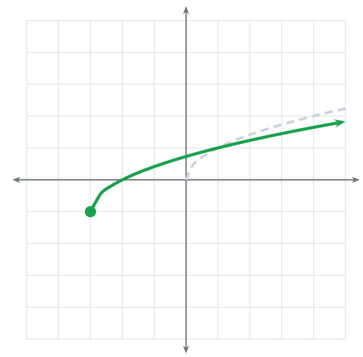
The gray dashed curve is always the parent; the **colored** curve is the transformed function with its key point marked. The same moves — shift, reflect, stretch — work on every family, so once you can read one, you can read them all.

Absolute value $y = |x| \rightarrow -|x+1| + 4$



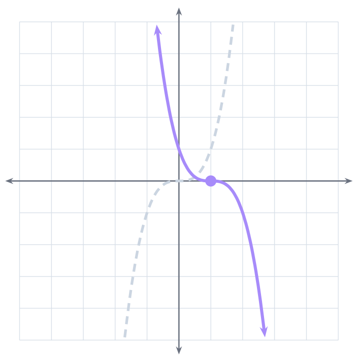
Reflect down, left 1, up 4 — vertex $(-1, 4)$.

Square root $y = \sqrt{x} \rightarrow \sqrt{x+3} - 1$



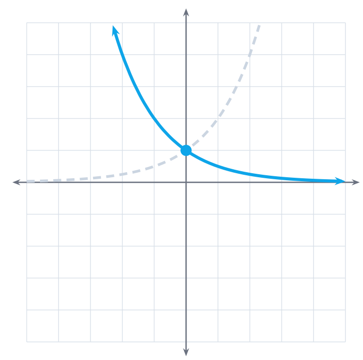
Left 3, down 1 — starts at $(-3, -1)$.

Cubic $y = x^3 \rightarrow -(x-1)^3$



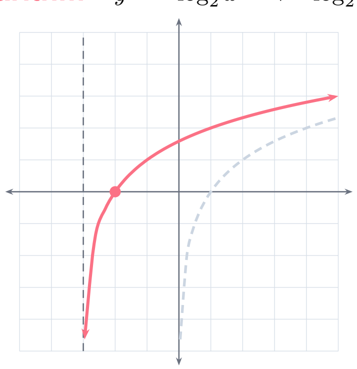
Reflect, right 1 — inflection at $(1, 0)$.

Exponential $y = 2^x \rightarrow 2^{-x}$



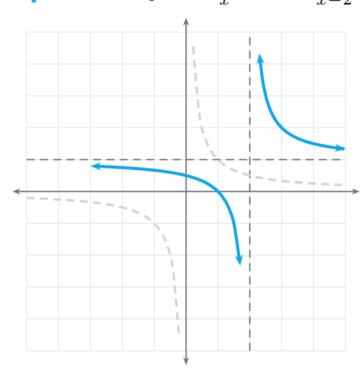
Reflect over the y-axis — same asymptote $y = 0$.

Logarithm $y = \log_2 x \rightarrow \log_2(x+3)$



Left 3 — asymptote moves to $x = -3$.

Reciprocal $y = \frac{1}{x} \rightarrow \frac{1}{x-2} + 1$



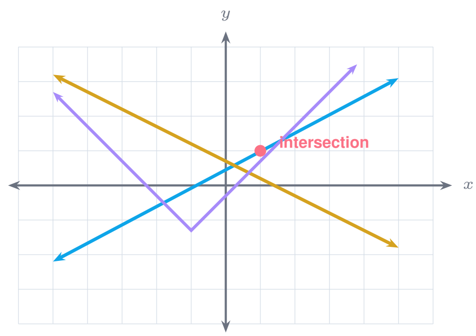
Right 2, up 1 — asymptotes $x = 2$, $y = 1$.

3 Linear, Absolute Value & Systems

Quick visual: line, V-shape, or intersection?

Lines track constant rate, absolute value graphs measure distance from a center, and systems ask where two rules agree. Before solving, sketch the story in one sentence.

Slope $m = \dfrac{\text{change in } y}{\text{change in } x}$ **System** shared point



Lines and systems

Slope	$m = \dfrac{y_2 - y_1}{x_2 - x_1}$, with $x_2 \neq x_1$.
Slope-intercept form	$y = mx + b$.
Point-slope form	$y - y_1 = m(x - x_1)$.
Standard form	$Ax + By = C$, usually with integer coefficients.
Parallel lines	Same slope.
Perpendicular lines	Slopes multiply to -1, unless one line is vertical and the other is horizontal.
Absolute value equation	$\lvert x - a\rvert = b$ gives $x = a \pm b$ when $b \geq 0$. If $b < 0$, no solution.
System types	One intersection: one solution. Parallel distinct lines: no solution. Same line: infinitely many solutions.

Tutor's Note

Systems ask for values that make every equation true at the same time. Substitution is great when one variable is already isolated. Elimination is great when coefficients can be made opposites.

Example

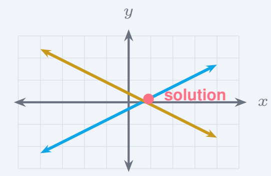
Solve $\begin{cases} 2x + y = 7 \\ x - y = 2 \end{cases}$. Add the equations to get $3x = 9$, so $x = 3$. Then $3 - y = 2$, so $y = 1$. The solution is $(3, 1)$.

INEQUALITIES

When multiplying or dividing an inequality by a negative number, reverse the inequality symbol. For $\lvert x - a\rvert < b$, write $a - b < x < a + b$. For $\lvert x - a\rvert > b$, write $x < a - b$ or $x > a + b$.

System story check

Before solving, predict the answer count: crossing lines mean **one** solution, parallel lines mean **none**, and the same line means **infinitely many**. Choose substitution when a variable is isolated; choose elimination when coefficients can cancel.



Intersection = shared answer

System solving choice Is one variable already alone? Use substitution. Do matching coefficients appear? Use elimination. Are the equations in graph form? Compare slopes and intercepts first.

4 Quadratics & Complex Numbers

Quadratic toolbox

Standard form $f(x) = ax^2 + bx + c$, where $a \neq 0$.

Vertex & axis $h = -\dfrac{b}{2a}$, $k = f(h)$; axis of symmetry $x = h$.

Vertex form $f(x) = a(x-h)^2 + k$. Opens up if $a > 0$ (min), down if $a < 0$ (max).

Factored form $f(x) = a(x - r_1)(x - r_2)$ shows zeros r_1, r_2.

Quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Discriminant $D = b^2 - 4ac$: $D > 0$ two real roots, $D = 0$ one real double root, $D < 0$ two complex roots.

Complex unit $i^2 = -1$, so $\sqrt{-a} = i\sqrt{a}$ for $a > 0$.

Powers of i cycle every 4: $i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$, then repeat.

Complex conjugate The conjugate of $a + bi$ is $a - bi$; $(a+bi)(a-bi) = a^2 + b^2$.

Tutor's Note

The same quadratic can be useful in different forms. Standard form is good for the quadratic formula and the y-intercept. Vertex form is best for maximum/minimum. Factored form is best for zeros.

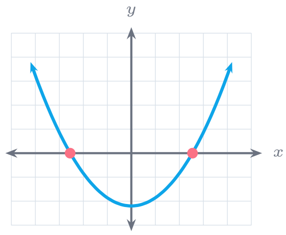
Example

For $f(x) = 2x^2 - 8x + 3$, the vertex has $h = -\frac{-8}{2(2)} = 2$ and $k = f(2) = 8 - 16 + 3 = -5$. Vertex: $(2, -5)$.

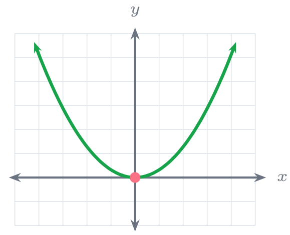
COMPLEX

When $D < 0$, the graph has no real x-intercepts, but the equation still has complex solutions. Keep $i^2 = -1$ handy when simplifying.

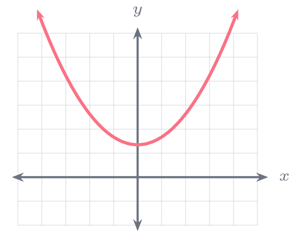
Discriminant snapshot Let $D = b^2 - 4ac$. It tells how many real x-intercepts the parabola has before you solve.



$D > 0$: two



$D = 0$: one



$D < 0$: none real

5 Polynomial Functions & Factoring

Polynomial formulas

Degree	Highest exponent after the polynomial is simplified.
Square of a binomial	$(a \pm b)^2 = a^2 \pm 2ab + b^2$.
Cube of a binomial	$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$; $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$.
Difference of squares	$a^2 - b^2 = (a+b)(a-b)$.
Sum/difference of cubes	$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$; $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$.
Division algorithm	$P(x) = D(x)\,Q(x) + R(x)$, with $\deg R < \deg D$ and $D(x) \neq 0$.
Remainder theorem	When $P(x)$ is divided by $x - c$, the remainder is $P(c)$. For $ax - b$, it is $P(\frac{b}{a})$.
Factor theorem	$x - c$ is a factor of $P(x)$ exactly when $P(c) = 0$.
Rational root candidates	For integer coefficients, candidates are $\pm\frac{p}{q}$ where p divides the constant and q divides the leading coefficient.
Fundamental theorem	A degree-n polynomial has exactly n complex roots, counting multiplicity.
Multiplicity	Even multiplicity: graph is tangent to (bounces off) the x-axis. Odd multiplicity: graph crosses, flattening if > 1.
End behavior	The leading term $a_n x^n$ controls the far left and far right ends.

Tutor's Note

Factoring is reverse multiplication. First take out a GCF, then look for special patterns, grouping, or trinomial factoring. For higher-degree polynomials, test possible rational zeros and use division to reduce the degree.

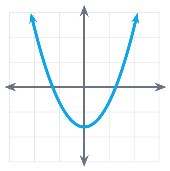
Example

Factor $x^3 - 8$. This is a difference of cubes: $x^3 - 2^3 = (x-2)(x^2 + 2x + 4)$.

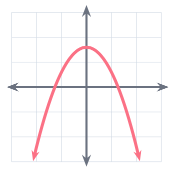
END BEHAVIOR

Even degree polynomials point the same way on both ends. Odd degree polynomials point opposite ways. A positive leading coefficient makes the right end go up; a negative leading coefficient makes the right end go down.

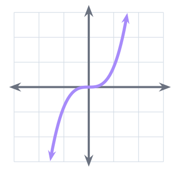
End behavior: degree & leading coefficient Only the leading term $a_n x^n$ controls the far ends. Even degree: both ends match. Odd degree: ends go opposite ways. The sign of a_n sets the right end.



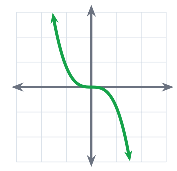
Even, $a_n > 0$



Even, $a_n < 0$



Odd, $a_n > 0$



Odd, $a_n < 0$

6 Rational Expressions & Variation

Rational expression rules

Restriction	Any value that makes a denominator 0 is not allowed.
Multiply	$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$, with $b, d \neq 0$.
Divide	$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$, with $b, c, d \neq 0$.
Add/subtract	$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$, with $b, d \neq 0$.
Direct variation	$y = kx$.
Inverse variation	$y = \frac{k}{x}$ or $xy = k$, with $x \neq 0$.
Joint variation	$y = kxz$.
Combined variation	$y = \frac{kx}{z}$, with $z \neq 0$.

Tutor's Note

Simplify rational expressions by factoring first, then cancel common factors. Never cancel pieces across addition or subtraction. For rational equations, multiply by the LCD, solve, then check every answer in the original equation.

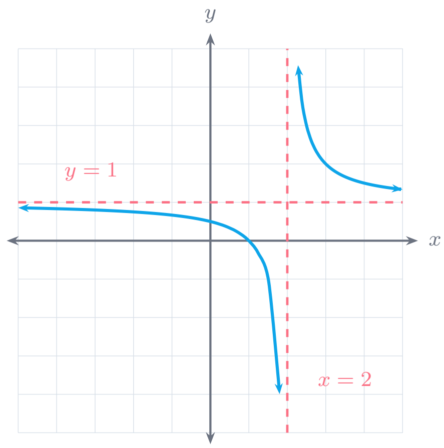
Example

$$\frac{x^2 - 9}{x^2 - 3x} = \frac{(x-3)(x+3)}{x(x-3)} = \frac{x+3}{x},$$ but the original restrictions are $x \neq 0$ and $x \neq 3$.

ASYMPTOTES

Vertical asymptotes often come from denominator zeros that remain after simplifying. Horizontal asymptotes compare degrees: lower numerator degree gives $y = 0$; equal degrees give the ratio of leading coefficients.

Reading a rational graph Denominator zeros that survive give *vertical asymptotes*; comparing degrees gives the *horizontal asymptote*; a canceled factor leaves a *hole*.



Vertical asymptote: set the surviving denominator to 0 ($x = 2$ here).

Horizontal asymptote: compare degrees — bottom-heavy gives $y = 0$, equal degrees give the ratio of leading coefficients, top-heavy gives none (slant instead).

Hole: a factor that cancels from top and bottom removes a single point, not a whole line.

7 Radicals & Rational Exponents

Laws of exponents

Product of powers $a^m \cdot a^n = a^{m+n}$.

Quotient of powers $\frac{a^m}{a^n} = a^{m-n}$, with $a \neq 0$.

Power of a power $(a^m)^n = a^{mn}$.

Power of a product/quotient $(ab)^n = a^n b^n$ and $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, with $b \neq 0$.

Zero exponent $a^0 = 1$, with $a \neq 0$.

Negative exponent $a^{-n} = \frac{1}{a^n}$ and $\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$, with nonzero bases.

Radical and exponent forms

Rational exponent $a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$ when the real expression is defined.

Product rule $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$ when both sides are real.

Quotient rule $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, with $b \neq 0$.

nth root of a^n $\sqrt[n]{a^n} = a$ if n is odd; $\sqrt[n]{a^n} = |a|$ if n is even.

Even roots $\sqrt{x}$ means the principal, nonnegative square root. For real even roots, the radicand must be nonnegative.

Conjugates $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$.

Radical equations Isolate the radical, raise both sides to the index power, solve, then check.

Tutor's Note

Radicals and rational exponents are two languages for the same idea. Use exponent rules when powers are easier; use radical notation when the root structure is clearer.

Example

$\frac{5}{\sqrt{3}}$ is rationalized by multiplying by $\frac{\sqrt{3}}{\sqrt{3}}$: $\frac{5\sqrt{3}}{3}$.

CHECK

Squaring both sides can create extraneous solutions. Always plug radical equation answers into the original equation.

8 Exponential & Logarithmic Functions

Growth, decay, and logs

Exponential model	$y = ab^x$, where a is the starting value and b is the multiplier.
Growth/decay	$A = P(1 + r)^t$ for growth; $A = P(1 - r)^t$ for decay.
Compound interest	$A = P\left(1 + \frac{r}{n}\right)^{nt}$.
Continuous growth	$A = Pe^{rt}$; population $N(t) = N_0e^{rt}$.
Log definition	$\log_b x = y \iff b^y = x$, where $b > 0$, $b \neq 1$, and $x > 0$.
Common & natural log	$\log x = \log_{10} x$ and $\ln x = \log_e x$.
Inverse properties	$\log_b 1 = 0$, $\log_b b = 1$, $b^{\log_b x} = x$, $\log_b(b^x) = x$.
Product rule	$\log_b(MN) = \log_b M + \log_b N$, where $M, N > 0$.
Quotient rule	$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$, where $M, N > 0$.
Power rule	$\log_b(M^p) = p\log_b M$, where $M > 0$.
Change of base	$\log_b M = \frac{\log_a M}{\log_a b} = \frac{\ln M}{\ln b}$, where $a, b > 0$ and $a, b \neq 1$.

Tutor's Note

Logs undo exponentials. If the variable is in the exponent, use logs. If the variable is inside a log, rewrite or condense logs until the equation can be solved.

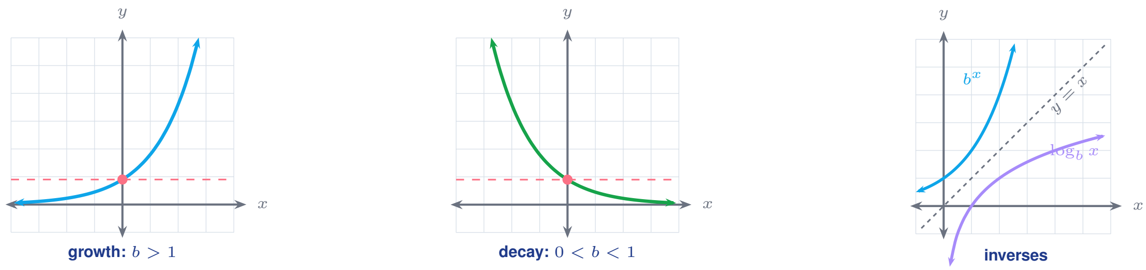
Example

Solve $3(2^x) = 24$. Divide by 3: $2^x = 8$. Since $8 = 2^3$, $x = 3$.

LOG DOMAIN

Every logarithm needs a positive argument. Check the original log equation, not just the simplified one.

Exponential & logarithmic visual atlas Exponentials change by a constant *multiplier*; a log is the inverse, so its graph is the mirror of the exponential across the line $y = x$.



$y = ab^x$ rises faster

$y = ab^x$ shrinks to 0

reflection across $y = x$

9 Sequences, Series & the Binomial Theorem

Sequence and series formulas

Arithmetic sequence $a_n = a_1 + (n-1)d.$

Arithmetic series $S_n = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}\big(2a_1 + (n-1)d\big).$

Geometric sequence $a_n = a_1 r^{n-1}.$

Finite geometric series $S_n = \frac{a_1(1-r^n)}{1-r}$, where $r \neq 1.$

Infinite geometric series $S_\infty = \frac{a_1}{1-r}$, only when $|r| < 1.$

Sigma notation $\sum_{k=1}^{n} a_k = a_1 + a_2 + \cdots + a_n.$

Combinations $\binom{n}{k} = \frac{n!}{k!(n-k)!}.$

Binomial theorem $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k.$

Tutor's Note

Arithmetic patterns add the same number each time. Geometric patterns multiply by the same number each time. Before using a formula, decide which pattern you have.

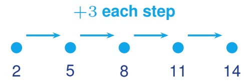
Example

For $4, 12, 36, \ldots$, $a_1 = 4$ and $r = 3$, so $a_6 = 4 \cdot 3^5 = 972.$

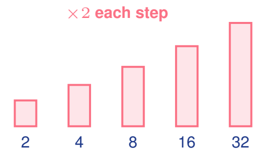
SERIES

An infinite geometric series has a finite sum only when the terms shrink toward zero, which means $|r| < 1.$

Arithmetic vs. geometric at a glance Arithmetic sequences *add* the same step d; geometric sequences *multiply* by the same ratio r. A series is the sum of those terms.



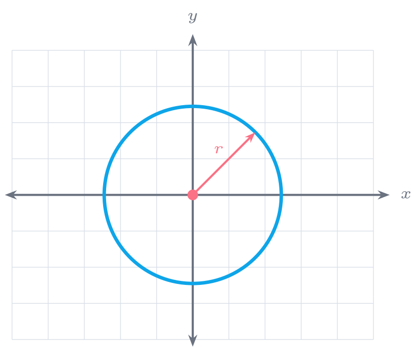
Arithmetic $a_n = a_1 + (n-1)d$



Geometric $a_n = a_1 r^{n-1}$

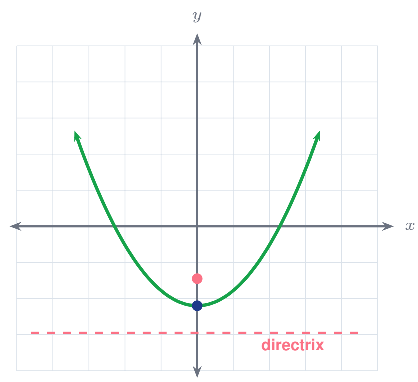
10 Conic Sections & Coordinate Formulas

Conic visual atlas Read a conic by asking: how many squared variables, plus or minus, and where are the center, vertex, foci, or asymptotes?



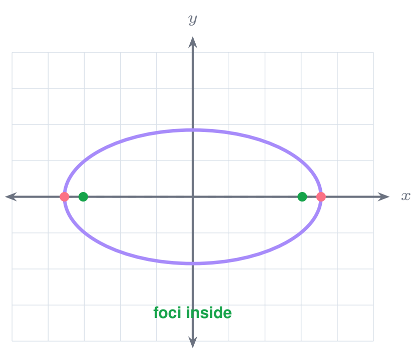
circle: center + radius

$$(x-h)^2 + (y-k)^2 = r^2$$



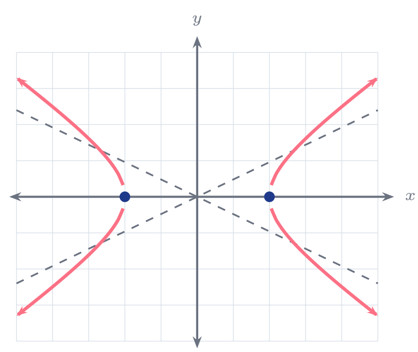
parabola: vertex + focus

$$(x-h)^2 = 4p(y-k)$$



ellipse: plus, closed oval

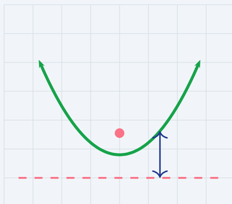
$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$



hyperbola: minus + asymptotes

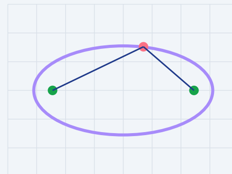
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Why the shapes happen Conics come from distance rules. The equation is easier to remember when the distance story is visible.



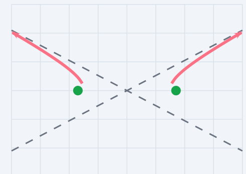
parabola

same distance to focus and directrix



ellipse

sum of distances to foci is constant



hyperbola

difference of distances to foci is constant

Coordinate and conic formulas

Distance	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
Midpoint	$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.
Circle	$(x-h)^2 + (y-k)^2 = r^2$. Center (h,k), radius r.
Vertical parabola	$(x-h)^2 = 4p(y-k)$. Focus $(h, k+p)$, directrix $y = k - p$.
Horizontal parabola	$(y-k)^2 = 4p(x-h)$. Focus $(h+p, k)$, directrix $x = h - p$.
Ellipse	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$. The larger denominator gives the major axis.
Ellipse foci	If $a \geq b$, $c^2 = a^2 - b^2$ and foci are $(h \pm c, k)$. If $b > a$, foci are $(h, k \pm c)$.
Horizontal hyperbola	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$. Vertices $(h \pm a, k)$, foci $(h \pm c, k)$.
Vertical hyperbola	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$. Vertices $(h, k \pm a)$, foci $(h, k \pm c)$.
Hyperbola facts	$c^2 = a^2 + b^2$. Horizontal asymptotes: $y - k = \pm\frac{b}{a}(x-h)$. Vertical asymptotes: $y - k = \pm\frac{a}{b}(x-h)$.

CIRCLE
Both squared terms have equal coefficients; one radius.

PARABOLA
Only one variable is squared; p controls focus direction.

ELLIPSE
Two squared terms are added; foci stay inside the oval.

HYPERBOLA
Squared terms are subtracted; branches chase asymptotes.

Tutor's Note

Conics are graphs made from distances. The center or vertex tells you where the graph lives; the denominators and the value of p tell you how it opens or stretches.

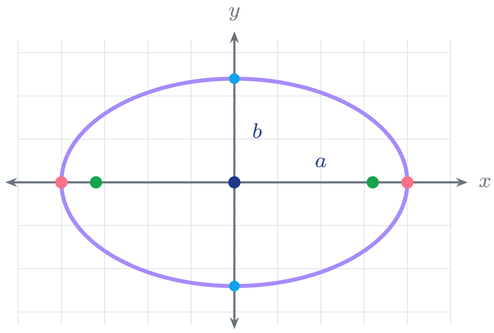
Example

$(x-2)^2 + (y+3)^2 = 25$ is a circle with center $(2, -3)$ and radius 5.

CONICS

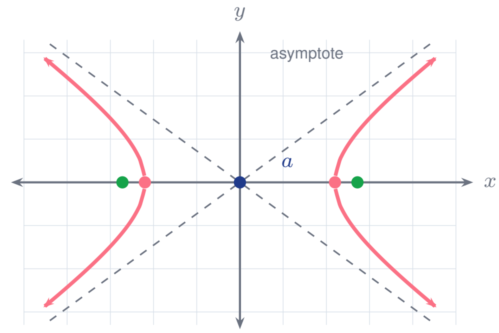
For ellipses, $c^2 = a^2 - b^2$. For hyperbolas, $c^2 = a^2 + b^2$. That sign change is one of the easiest places to lose accuracy.

Anatomy of an ellipse & a hyperbola The same parts appear in both: a center, two vertices, and two foci on the main axis. Only the sign and the focus rule change.



• center • vertex • focus • co-vertex

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1, \; c^2 = a^2 - b^2$

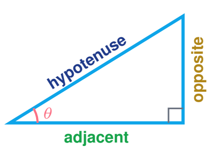


• center • vertex • focus

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1, \; c^2 = a^2 + b^2$

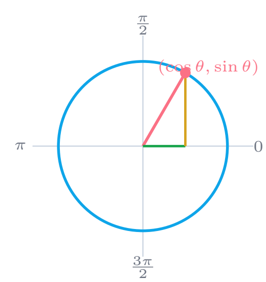
11 Trigonometry Essentials

Trigonometry visual atlas Trig connects a right triangle, the unit circle, radian measure, and repeating wave graphs. These are the same ratios seen four ways.

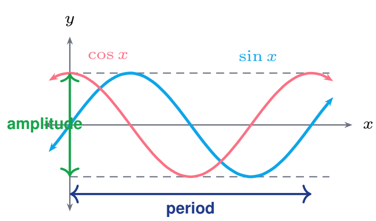


SOH-CAH-TOA

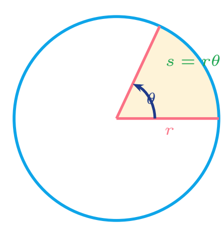
$\sin\theta = \frac{o}{h}, \quad \cos\theta = \frac{a}{h}, \quad \tan\theta = \frac{o}{a}$



unit circle: $x = \cos\theta, y = \sin\theta$



$y = A\sin(B(x - C)) + D$: amplitude $|A|$, period $\frac{2\pi}{|B|}$



radians measure arc per radius

$A_{\text{sector}} = \frac{1}{2}r^2\theta, \quad 180^\circ = \pi$ radians

Trig formulas

Coterminal angles	$\theta + 2\pi k$ radians or $\theta + 360^\circ k$, where k is any integer.
Radians/degrees	$\theta_{\text{rad}} = \theta_{\text{deg}}\dfrac{\pi}{180}$ and $\theta_{\text{deg}} = \theta_{\text{rad}}\dfrac{180}{\pi}$.
Right triangle ratios	$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$, $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$, $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$.
Unit circle	$x = \cos\theta$, $y = \sin\theta$, and $\tan\theta = \frac{\sin\theta}{\cos\theta}$ when $\cos\theta \neq 0$.
Pythagorean identity	$\sin^2\theta + \cos^2\theta = 1$.
Reciprocal identities	$\csc\theta = \frac{1}{\sin\theta}$, $\sec\theta = \frac{1}{\cos\theta}$, $\cot\theta = \frac{1}{\tan\theta}$.
Quotient identities	$\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$ when defined.
Trig graph	$y = A\sin(B(x-C)) + D$ has amplitude $\lvert A\rvert$, period $\frac{2\pi}{\lvert B\rvert}$, phase shift C, midline $y = D$.
Frequency	frequency $= \dfrac{1}{\text{period}} = \dfrac{\lvert B\rvert}{2\pi}$ — the number of cycles per unit.
Arc length and sector	$s = r\theta$ and $A = \frac{1}{2}r^2\theta$ when θ is in radians.
Law of sines	$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.
Law of cosines	$c^2 = a^2 + b^2 - 2ab\cos C$.
Triangle area	$K = \frac{1}{2}ab\sin C$ when sides a and b include angle C.

Special-angle values

Memorize quadrant I, then use signs by quadrant.

Angle	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

Tutor's Note

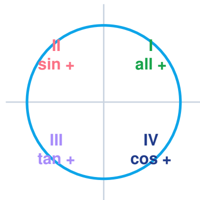
Trig connects angles, circles, and waves. In right triangles, use SOH-CAH-TOA. On the unit circle, cosine is the x-coordinate and sine is the y-coordinate.

Example

If $\theta = \frac{\pi}{3}$, then $\sin\theta = \frac{\sqrt{3}}{2}$ and $\cos\theta = \frac{1}{2}$.

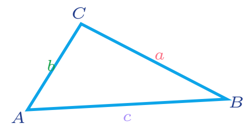
RADIANS

Arc length, sector area, and trig graph periods use radians unless the problem clearly says degrees.



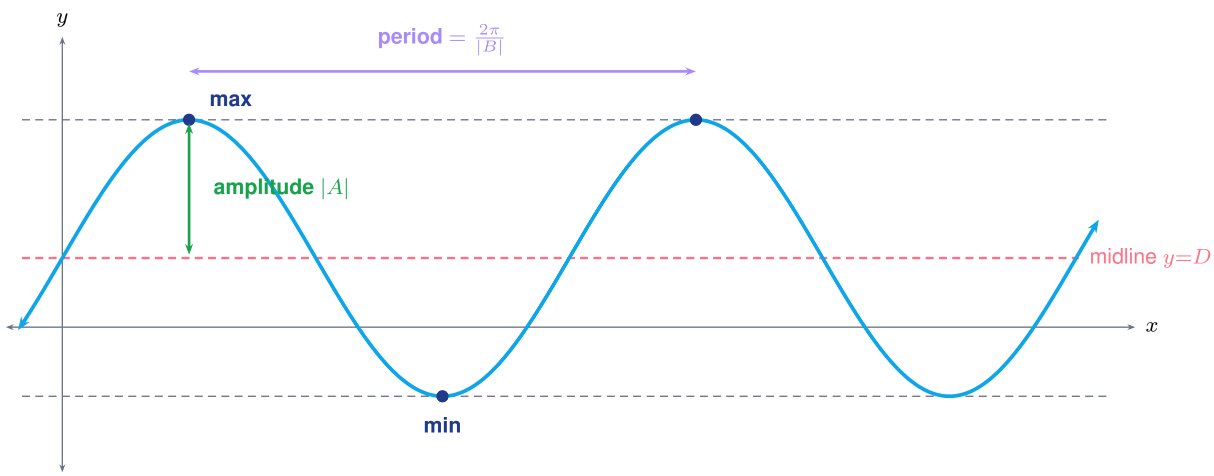
Choosing a trig tool

Use the unit circle for exact values and signs. Use the graph form for amplitude, period, shifts, and midline. Use the Law of Sines or Cosines when a triangle is not right.



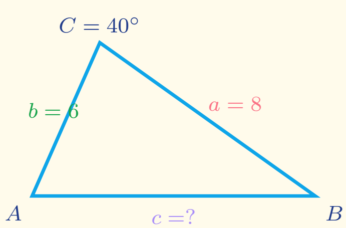
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Anatomy of a sinusoid: $y = A\sin\big(B(x - C)\big) + D$ Amplitude $|A|$ is the height from the midline to a peak; the midline is $y = D$; the period $\frac{2\pi}{|B|}$ is one full cycle; C slides the wave sideways.



Solving a non-right triangle

Example



Two sides and the included angle are known, so start with the **Law of Cosines**:

$c^2 = a^2 + b^2 - 2ab\cos C = 8^2 + 6^2 - 2(8)(6)\cos 40°$

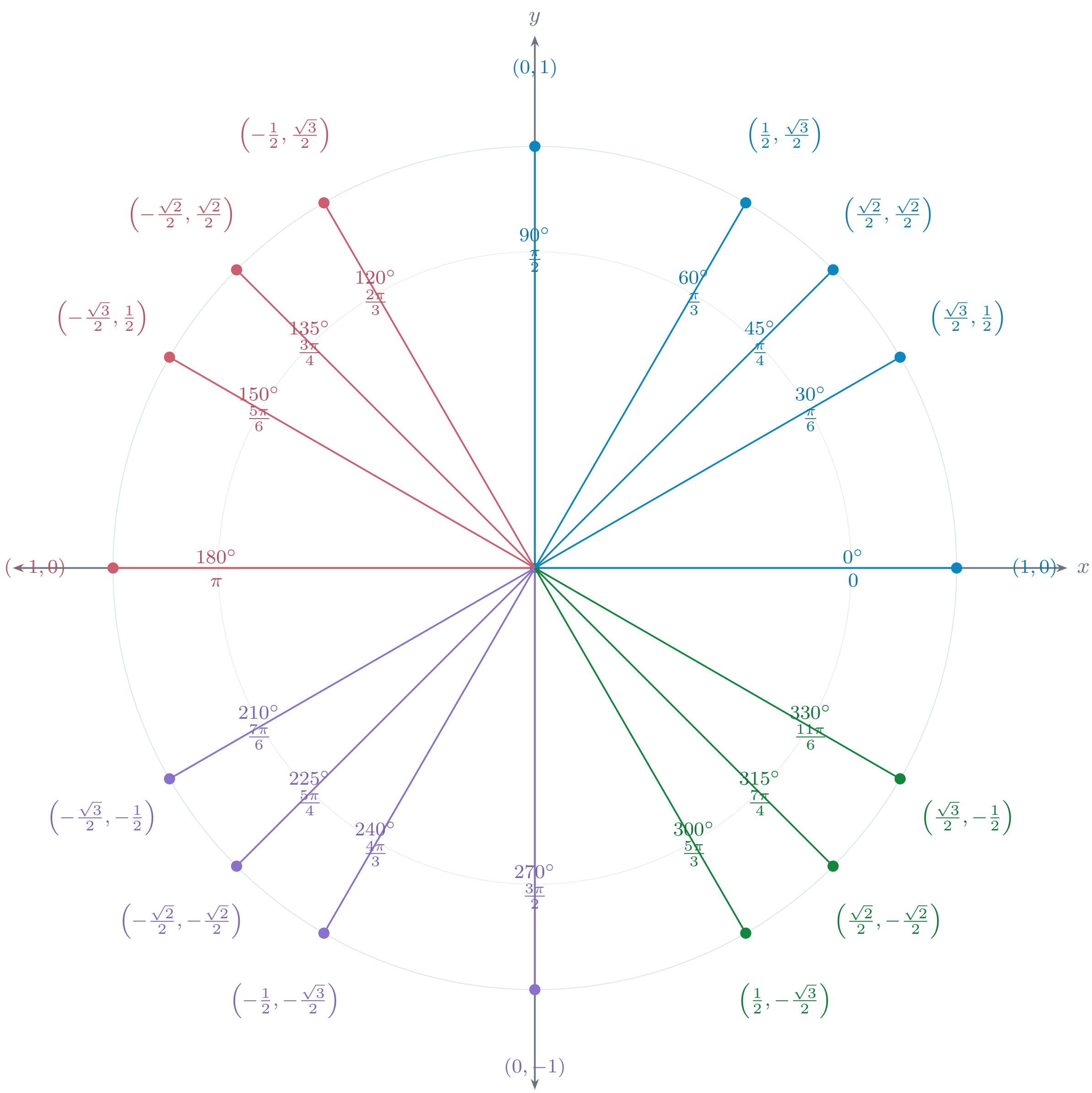
$c^2 = 100 - 96(0.766) \approx 26.5$, so $c \approx 5.15$.

Then the **Law of Sines** finds an angle: $\dfrac{\sin A}{8} = \dfrac{\sin 40°}{5.15} \Rightarrow A \approx 87°$.

12 Trigonometry Reference: Unit Circle & Identities

The Unit Circle

Every special angle in one place: each point is $(\cos\theta, \sin\theta)$. Read the angle in degrees and radians inside, and its coordinates outside.



QUADRANT SIGNS

In quadrant I all ratios are positive. After that, only one family stays positive: **S**ine in II, **T**angent in III, **C**osine in IV ("All Students Take Calculus").

Key trigonometric identities

Pythagorean identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

Double-angle

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$$
$$\cos 2\theta = 1 - 2\sin^2\theta \qquad \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Sum & difference

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

Half-angle

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$
$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$
$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

Even & odd

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta$$
$$\tan(-\theta) = -\tan\theta$$

Cofunction & reciprocal

$$\sin\left(\tfrac{\pi}{2} - \theta\right) = \cos\theta \qquad \tan\left(\tfrac{\pi}{2} - \theta\right) = \cot\theta$$
$$\csc\theta = \frac{1}{\sin\theta}, \quad \sec\theta = \frac{1}{\cos\theta}, \quad \cot\theta = \frac{1}{\tan\theta}$$

Inverse trigonometric functions

$y = \sin^{-1} x$	Domain $-1 \le x \le 1$, Range $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$. $\sin(\sin^{-1} x) = x$.
$y = \cos^{-1} x$	Domain $-1 \le x \le 1$, Range $0 \le y \le \pi$. $\cos(\cos^{-1} x) = x$.
$y = \tan^{-1} x$	Domain all real x, Range $-\frac{\pi}{2} < y < \frac{\pi}{2}$. $\tan(\tan^{-1} x) = x$.
Watch out	$\sin^{-1} x$ is an *angle*, not $\frac{1}{\sin x}$. The reciprocal is $(\sin x)^{-1} = \csc x$.

Tutor's Note

You only need to memorize a few of these. The Pythagorean and reciprocal identities come straight from the unit circle, and the double-angle formulas are just the sum formulas with $A = B$. Derive the rest when you need them.

VERIFYING

To prove an identity, work on the more complicated side only and rewrite everything in terms of sin and cos. Do not move terms across the equals sign as if solving an equation.

13 Matrices

Matrix operations

Dimensions — An $m \times n$ matrix has m rows and n columns.

Addition/subtraction — Only matrices with the same dimensions can be added or subtracted.

Scalar multiplication — Multiply every entry by the scalar.

Matrix multiplication — AB is defined when columns of A equal rows of B. The result has rows of A and columns of B.

2×2 determinant — $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc.$

2×2 inverse — $A^{-1} = \dfrac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, if $ad - bc \neq 0$.

Matrix equation — $AX = B$ gives $X = A^{-1}B$ when A^{-1} exists.

Tutor's Note

Matrix multiplication is not entry-by-entry. Each entry in the product comes from a row times a column. Also, order matters: usually $AB \neq BA$.

Example

For $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $\det(A) = 1 \cdot 4 - 2 \cdot 3 = -2$, so the matrix has an inverse.

INVERSE

A matrix with determinant 0 is singular, so it has no inverse.

How matrix multiplication works

Each entry of AB is a *row of A* dotted with a *column of B*: multiply matching pieces, then add.

$$\underset{A}{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}} \times \underset{B}{\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

$(1)(5) + (2)(7) = 19$

Row 1 of A meets column 1 of B. Inner dimensions must match; the answer keeps the outer dimensions.

14 Statistics & Probability

Data and chance formulas

Mean	$\bar{x} = \frac{\sum x}{n}.$
Weighted mean	$\frac{\sum wx}{\sum w}.$
Population variance	$\sigma^2 = \frac{\sum (x-\mu)^2}{N}.$
Sample variance	$s^2 = \frac{\sum (x-\bar{x})^2}{n-1}.$
Z-score	$z = \frac{x-\mu}{\sigma}.$
Permutation	${}_nP_r = \frac{n!}{(n-r)!}.$
Combination	${}_nC_r = \frac{n!}{r!(n-r)!}.$
Addition rule	$P(A \cup B) = P(A) + P(B) - P(A \cap B).$
Conditional probability	$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$, with $P(B) > 0.$
Binomial probability	$P(k) = \binom{n}{k} p^k (1-p)^{n-k}.$

Tutor's Note

Choose permutations when order matters and combinations when order does not. For probability, make sure you know whether events overlap, are independent, or are being conditioned on another event.

Example

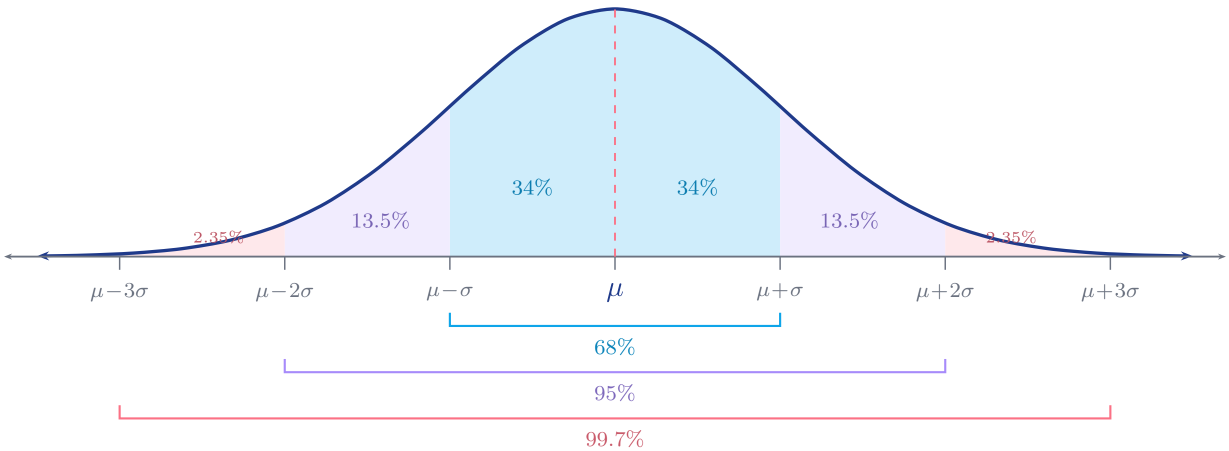
From 8 students, the number of ways to choose 3 for a committee is $\binom{8}{3} = \frac{8!}{3!5!} = 56.$

DATA

Correlation r is between -1 and 1. A strong correlation does not prove that one variable causes the other.

The normal curve & the empirical rule

For bell-shaped data, a z-score counts how many standard deviations a value sits from the mean. About 68%, 95%, and 99.7% of the data falls within 1, 2, and 3 standard deviations of the mean μ.



A z-score turns any normal value into a position on this curve. Positive z sits right of the mean, negative z sits left, and $z = 0$ is the mean itself.

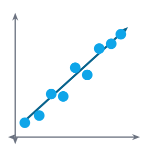
$$z = \frac{x-\mu}{\sigma}$$

Example

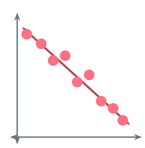
A test has mean $\mu = 70$ and standard deviation $\sigma = 8$. A score of 86 gives $z = \frac{86-70}{8} = 2.0$ — two deviations *above* the mean, beating about 97.5% of scores. A score of 62 gives $z = \frac{62-70}{8} = -1.0$, one deviation below.

Correlation at a glance

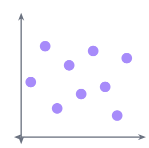
The correlation r (between -1 and 1) measures how tightly points hug a line. Strong correlation is *not* the same as causation.



positive, $r \approx +0.9$



negative, $r \approx -0.9$



none, $r \approx 0$

Algebra 2 Symbols Cheat Sheet

Symbol • Meaning • Example

Functions & Graphs

Symbol	Meaning	Example
$f(x)$	output of function f	$f(2) = 7$
$f^{-1}(x)$	inverse function	$f^{-1}(5) = 2$
$f \circ g$	composition	$(f \circ g)(x)$
Δ	change in	$\Delta y/\Delta x$
D	discriminant	$D = b^2 - 4ac$
i	imaginary unit	$i^2 = -1$
$\|z\|$	complex modulus	$\|3 + 4i\| = 5$
m	slope	$y = mx + b$
b	y-intercept	$y = 2x + b$
(x, y)	ordered pair	$(3, -1)$
$\sim$	similar / same shape	$\triangle A \sim \triangle B$

Trig & Conics

Symbol	Meaning	Example
θ	angle measure	$\theta = \pi/6$
π	half turn in radians	$180^\circ = \pi$
$\circ$	degrees	90°
$\angle$	angle	$\angle A$
$\sin, \cos, \tan$	trig ratios	$\sin\theta = o/h$
$\sec, \csc, \cot$	reciprocal trig ratios	$\sec\theta = 1/\cos\theta$
(h, k)	center or vertex	circle center (h, k)
r	radius	$(x - h)^2 + (y - k)^2 = r^2$
p	directed focus distance	$(x - h)^2 = 4p(y - k)$
a, b, c	conic parameters	$c^2 = a^2 - b^2$ for ellipse

Sets, Logs & Sequences

Symbol	Meaning	Example
$\mathbb{R}$	real numbers	$x \in \mathbb{R}$
$\mathbb{C}$	complex numbers	$2 - 3i \in \mathbb{C}$
$\mathbb{Z}$	integers	$\ldots, -1, 0, 1, \ldots$
$\in$	is an element of	$3 \in \mathbb{R}$
$\notin$	is not an element of	$\sqrt{2} \notin \mathbb{Z}$
$\ln x$	natural log	$\ln e = 1$
$\log_b x$	logarithm base b	$\log_2 8 = 3$
$\sum$	sum	$\sum_{k=1}^{4} k = 10$
a_n	nth term	$a_n = 3n + 1$
$n!$	factorial	$5! = 120$
$\binom{n}{r}$	combinations	$\binom{5}{2} = 10$

Probability & Matrices

Symbol	Meaning	Example
$P(A)$	probability of event A	$P(A) = 0.4$
$P(A \mid B)$	probability of A given B	$P(A \mid B)$
$\bar{x}$	sample mean	$\bar{x} = \sum x/n$
μ	population mean	center of data
σ	population std. deviation	spread of data
z	standard score	$z = (x - \mu)/\sigma$
$A \cup B$	A or B	union
$A \cap B$	A and B	intersection
A^{-1}	inverse matrix	$AA^{-1} = I$
I	identity matrix	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
$\det(A)$	determinant	$ad - bc$
s	sample std. deviation	sample spread

How to read symbols fast First identify the action ($=$, $<$, $\sum$, $\log$), then the allowed set ($\mathbb{R}$, $\mathbb{C}$, interval, or matrix size), then the restriction.

Algebra 2 Vocabulary Bank

Functions & Equations

Domain — all allowed inputs of a relation or function.
Range — all possible outputs.
Composition — using one function's output as another function's input.
Inverse function — a function that reverses another function.
Transformation — a shift, reflection, stretch, or compression of a graph.
Asymptote — a line a graph approaches but may not reach.
Extraneous solution — an answer produced by algebra steps that fails the original problem.
Restriction — a value not allowed by the original expression or equation.
One-to-one — a function where each output comes from only one input.
Average rate of change — the slope between two points on a function.
Piecewise function — a function with different rules on different intervals.
System of equations — two or more equations solved at the same time.

Logs, Sequences & Conics

Logarithm — the exponent needed to produce a number from a base.
Natural logarithm — a logarithm with base e.
Arithmetic sequence — a sequence with a common difference.
Geometric sequence — a sequence with a common ratio.
Series — a sum of sequence terms.
Conic section — a circle, parabola, ellipse, or hyperbola.
Focus — a fixed point used to define a conic.
Directrix — a fixed line used to define a parabola.
Eccentricity — a measure of how stretched a conic is.
Common difference — the repeated add-on in an arithmetic sequence.
Common ratio — the repeated multiplier in a geometric sequence.
Base — the repeated factor in an exponential or logarithm.

Polynomials, Rationals & Radicals

Polynomial — a sum of terms with whole-number exponents.
Multiplicity — how many times a factor repeats.
End behavior — what a graph does as x goes far left or far right.
Rational expression — a quotient of polynomials.
Radicand — the expression under a radical sign.
Conjugate — a binomial with the middle sign changed.
Rational exponent — an exponent that represents a root and a power.
Factor theorem — the rule connecting zeros and factors.
Synthetic division — a shortcut for dividing by $x - c$.
Leading coefficient — the coefficient on the highest-degree term.
Asymptote — a line a rational graph approaches.
Extraneous solution — a solved value that fails the original equation.

Trig, Matrices & Data

Radian — angle measure based on arc length and radius.
Reference angle — the acute angle to the x-axis.
Period — the length of one full cycle of a trig graph.
Amplitude — half the distance between maximum and minimum of a sinusoid.
Matrix — a rectangular array of numbers.
Determinant — a number that tells whether a square matrix has an inverse.
Correlation — the strength and direction of a linear relationship.
Conditional probability — probability after another event is known.
Binomial experiment — fixed trials with two outcomes and constant success probability.
Independent events — events where one result does not change the other probability.
Permutation — an arrangement where order matters.
Combination — a selection where order does not matter.

VOCAB

In Algebra 2, vocabulary often carries the restriction: inverse, log, rational, radical, and matrix inverse all require checking allowed values.

Vocabulary study loop Define it, sketch it, give one formula that uses it, then name the restriction. This turns vocabulary into problem-solving memory.

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