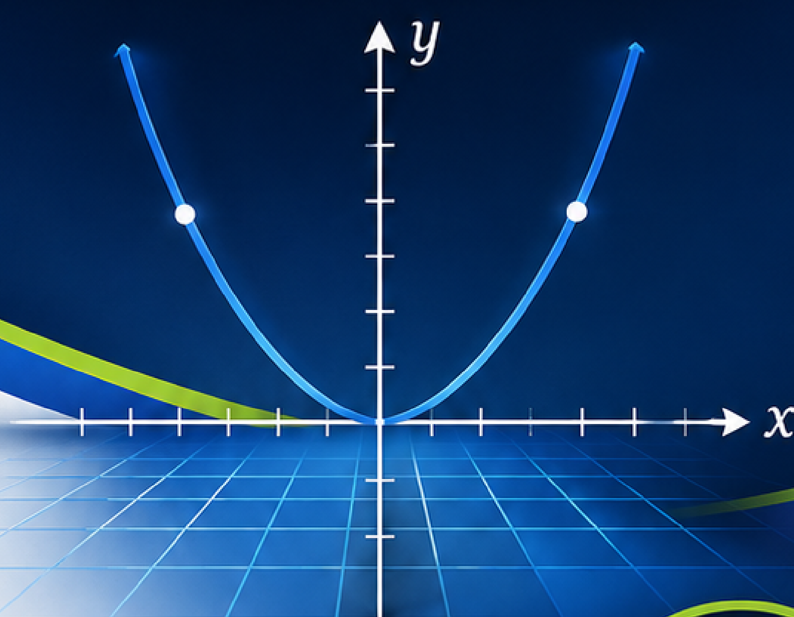
ALGEBRA 1

FORMULA REVIEW

AND

TUTORING GUIDE



ESSENTIAL FORMULAS
Quick reference for every key concept

STEP-BY-STEP EXAMPLES
Clear solutions to build your understanding

TUTORING SUPPORT
Tips, practice, and guidance to help you succeed

BUILD CONFIDENCE
Strengthen skills. Improve grades. Reach your goals.

REVIEW. PRACTICE. SUCCEED.

YOUR GUIDE TO MASTERING ALGEBRA 1

EffortlessMath.com

COMPLETE FORMULA REVIEW & TUTORING GUIDE

ALGEBRA 1

Every Formula, Clearly Explained

Rules, reasons, diagrams, and worked examples for the skills students use again and again in Algebra 1.

Formula Snapshot

LINEAR

$$y = mx + b$$

slope + intercept

QUADRATIC

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

works for every

$ax^2 + bx + c = 0, \ a \neq 0$

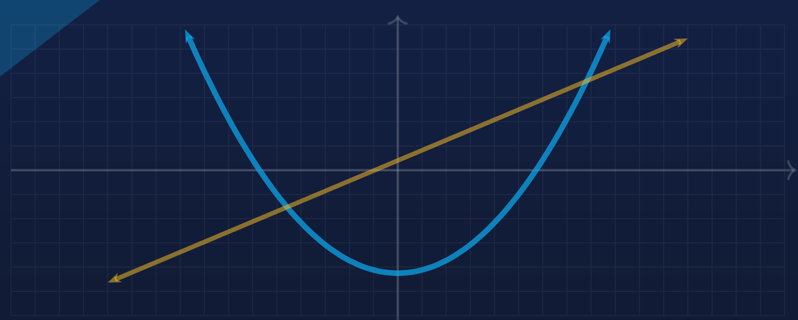
EXPONENTS

$$a^m a^n = a^{m+n}$$

same base

Quick Reference | Plain-English Tutor Notes | Worked Examples

Equations • Functions • Exponents • Polynomials • Quadratics • Radicals • Systems • Statistics



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Understand it. Practice it. Master it.

Hi there — let's make Algebra make sense.

Most formula sheets hand you a wall of symbols and wish you luck. This guide does something different. Think of it as sitting next to a patient tutor: for every formula you will get the rule itself, a plain-English explanation of *why* it works, and a worked example you can follow step by step. When you understand the reasoning behind a rule, you no longer have to memorize it — it just makes sense, and it stays with you.

Tutor's Note

How to use this guide Read a little at a time. For each idea, glance at the formula, then read the green "In Plain English" note like a tutor is talking you through it, then work the gold example yourself before reading the solution. If a topic feels shaky, slow down and re-read the explanation — that is exactly where the learning happens.

The color key

Blue: the formula

The rule itself, stated cleanly so you can find it fast.

Coral: a key rule or warning. The thing students most often forget — pay extra attention here.

Tutor's Note

Green: the tutor's voice The friendly "why it works" in everyday language.

Gold: a worked example. A real problem solved one step at a time.

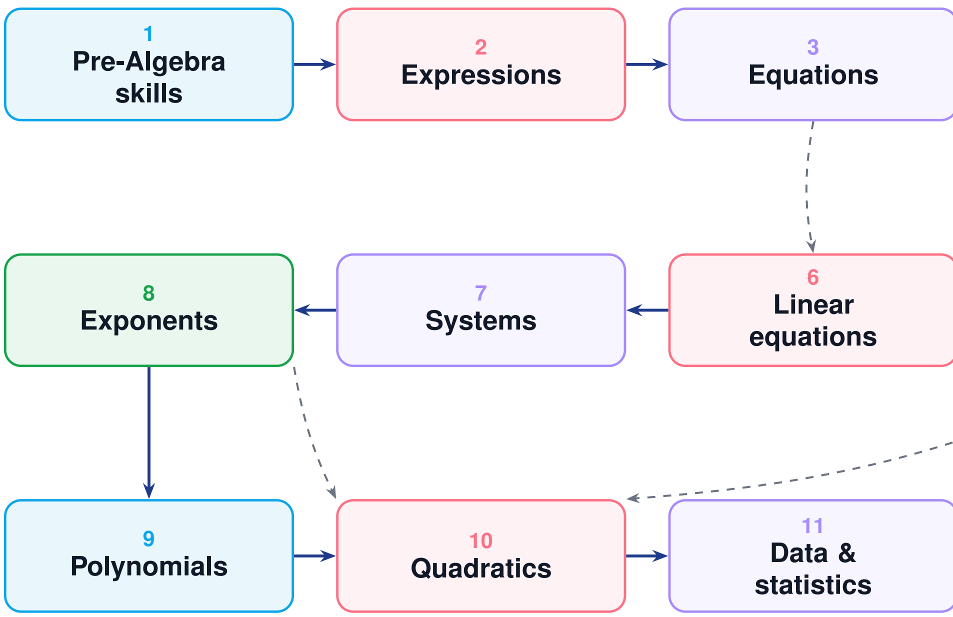
You've got this. Algebra is just a handful of big ideas, each built from the one before it. Take them one at a time, and by the last page the whole subject will feel like a story you already know.

What's Inside

Each section pairs the **formula** with a warm **Tutor's Note** that explains *why* it works — plus worked examples, key rules, and tips along the way.

Algebra Roadmap

> Algebra 1 is not a pile of separate rules. Each topic gives you a tool for the next one: numbers become expressions, expressions become equations, equations become graphs and models.



Big picture
Algebra is a language for patterns: write them, solve them, graph them, and use them to explain real data.

How to read the map

Follow the solid arrows for the main path. The dashed arrows show ideas that connect across chapters, such as equations turning into lines and functions.

Where students grow fastest

Master expressions, equation solving, and graph reading early. Those skills show up again in systems, polynomials, quadratics, and data questions.

ROADMAP When a topic feels hard, look one step earlier on the map. Most Algebra 1 mistakes come from a missing earlier skill, not from the new idea itself.

1 Foundations of Algebra

Before we build anything tall, we need a solid base. This section covers the ground rules that every later topic leans on — how numbers behave, the order we work in, and how to handle signs and fractions without fear.

The Real-Number Properties

These four properties are the "rules of the game." You already use them without thinking; algebra just gives them names.

Commutative (order)

$a + b = b + a$

$a \cdot b = b \cdot a$

Associative (grouping)

$(a + b) + c = a + (b + c)$

$(ab)c = a(bc)$

Distributive (share out)

$a(b + c) = ab + ac$

Identity & Inverse

$a + 0 = a, \quad a \cdot 1 = a$

$a + (-a) = 0, \quad a \cdot \frac{1}{a} = 1 \ (a \neq 0)$

Tutor's Note

Commutative means order doesn't matter for adding or multiplying: $3 + 5$ and $5 + 3$ both give 8. **Associative** means it doesn't matter how you group them: you can add the first two numbers or the last two and land in the same place. The **distributive** property is the real workhorse of algebra — it lets you "hand out" a multiplier to everything inside parentheses, which is how we expand and factor later. Notice these work for $+$ and $\times$, but *not* for subtraction or division: $6 - 2 \neq 2 - 6$.

Order of Operations (PEMDAS)

When an expression has several operations, we all need to agree on what to do first — otherwise the same expression could give different answers.

Do them in this order

Parentheses $\to$ **E**xponents $\to$ **M**ultiplication & **D**ivision (left $\to$ right) $\to$ **A**ddition & **S**ubtraction (left $\to$ right)

Tutor's Note

A useful phrase is "*Please Excuse My Dear Aunt Sally.*" The one trap: multiply and divide are a *team* — do whichever comes first reading left to right, not all multiplication before all division. Same goes for add and subtract.

Example

$3 + 4 \times 2^2 - (6 - 2)$. First parentheses: $(6 - 2) = 4$. Then the exponent: $2^2 = 4$. Then multiply: $4 \times 4 = 16$. Finally left to right: $3 + 16 - 4 = 15$.

Integer & Sign Rules

Signs trip up more students than almost anything else, so let's make them automatic.

Multiply / Divide

$(+)(+) = +$ $(-)(-) = +$

$(+)(-) = -$ $(-)(+) = -$

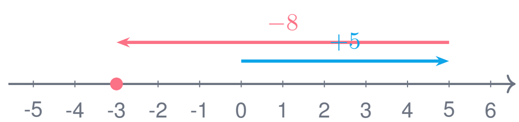
Add / Subtract

Same signs → add, keep the sign.
Different signs → subtract, keep the sign of the bigger number.
$a - b = a + (-b)$

Tutor's Note

For multiplying and dividing, here's the shortcut: count the negative signs. An *even* number of negatives gives a positive answer; an *odd* number gives a negative. For adding, picture a number line — same signs pile up in one direction, different signs partly cancel. And subtracting is just "adding the opposite," which is why $5 - 8$ is the same as $5 + (-8) = -3$.

Visual: adding on a number line



$5 + (-8) = -3$

Absolute Value

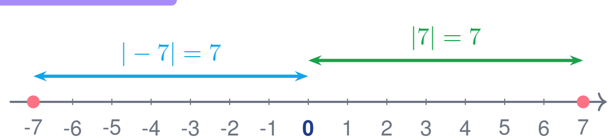
Distance from zero

$$|a| = \begin{cases} a & a \geq 0 \\ -a & a < 0 \end{cases} \qquad |a| \geq 0, \quad |a| = |-a|, \quad |ab| = |a|\,|b|$$

Tutor's Note

Absolute value answers one question: "how far from zero?" Distance is never negative, so $|-7| = 7$ and $|7| = 7$. That little "$-a$ when $a < 0$" line looks strange, but it just means: if the number is already negative, flip it positive.

Visual: absolute value is distance



Working with Fractions

For all fraction rules below, denominators cannot be zero. In division, the fraction you divide by also cannot equal zero.

$$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$$
$$\frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$$
$$\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$$
$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{ad}{bc}$$

Tutor's Note

To add or subtract, the denominators must match — that's what the cross-multiplying in the formula quietly does: it builds a common denominator bd. To multiply, just go straight across, top times top and bottom times bottom. To divide, "keep, change, flip": keep the first fraction, change $\div$ to $\times$, and flip the second one.

ZERO

If you try to divide by 0, there is no real value for the answer. In Algebra 1, write *undefined*. After fraction work, simplify by dividing the top and bottom by their greatest common factor.

Percent, Ratio & Proportion

Percent

part $=$ percent $\times$ whole (use the percent as a decimal)

$$\%\text{ change}=\frac{\text{new}-\text{old}}{\text{old}}\times 100 \quad (\text{old}\neq 0)$$

Proportion

$$\frac{a}{b}=\frac{c}{d}\implies ad=bc\quad (b,d\neq 0)$$

(cross-multiply to solve)

Tutor's Note

"Percent" literally means "per hundred," so 25% is just $\frac{25}{100}=0.25$. A proportion is two equal ratios; whenever you see one, cross-multiplying turns it into a simple equation you can solve. This single idea quietly powers unit rates, scale models, recipes, and similar triangles later on.

Example

What is 15% of 80? Part $=0.15\times 80=12$.

2 Linear Equations & Inequalities

An equation is a balance scale: whatever you do to one side, you must do to the other to keep it level. Solving means getting the variable alone on one side.

Solving Linear Equations

The goal: isolate the variable

Undo operations in reverse PEMDAS order.

$$ax + b = c \implies x = \frac{c-b}{a} \quad (a \neq 0)$$

Tutor's Note

Picture a wrapped present: it was wrapped in a certain order, and you unwrap it in the *opposite* order. If x was multiplied by a and then had b added, you undo it by first subtracting b, then dividing by a. Every step you do to the left side, do to the right side too — that keeps the scale balanced.

Example

$3x + 5 = 20$. Subtract 5: $3x = 15$. Divide by 3: $x = 5$. Check: $3(5) + 5 = 20$. ✓

Special Solution Cases

Sometimes the variable disappears as you solve. That's not a mistake — it's telling you something.

One solution
$2x + 1 = x + 4 \Rightarrow x = 3$

Infinitely many
$2(x + 1) = 2x + 2$ (always true)

No solution
$x + 2 = x + 5 \Rightarrow 2 = 5$ (false)

Literal equations
$A = \ell w \Rightarrow w = \frac{A}{\ell} \quad (\ell \neq 0)$

Tutor's Note

If you end with something *false* like $2 = 5$, no number can make it work, so there's *no solution*. If you end with something *always true* like $2 = 2$, then *every* number works. A "literal" equation just has more than one letter — you solve for one variable using the same balancing moves, treating the others like numbers.

Inequalities

The one new rule

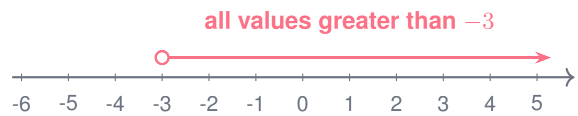
Solve exactly like an equation, **but flip the inequality sign** whenever you multiply or divide both sides by a *negative* number.

$$-2x < 6 \implies x > -3$$

Tutor's Note

Why the flip? Think about a true statement like $3 < 5$. Multiply both sides by -1 and you get -3 and -5. But -3 is actually *greater* than -5, so the sign has to turn around to stay true. That's the whole reason — negatives reverse order on the number line.

Visual: graphing $x > -3$



Reading the symbols

$<$ less than $>$ greater than
$\leq$ at most $\geq$ at least

Compound inequalities

AND: $a < x < b$ (between)
OR: $x < a$ or $x > b$

GRAPH A solid dot or closed bracket means "or equal to" ($\leq, \geq$). An open dot means strictly less or strictly greater.

Absolute-Value Equations & Inequalities

Split into two cases

$$|ax+b| = c \;\Rightarrow\; ax+b=c \text{ or } ax+b=-c \quad (c \geq 0)$$

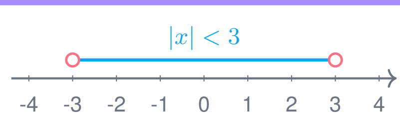
$$|x| < c \;\Rightarrow\; -c < x < c \qquad |x| > c \;\Rightarrow\; x < -c \text{ or } x > c \quad (c > 0)$$

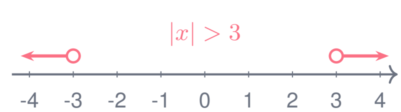
Tutor's Note

Because absolute value measures distance, $|x| = 5$ asks "what is 5 away from zero?" — and there are *two* answers, 5 and -5. So every absolute-value equation splits into two. For "less than," you want the points *close* to zero (a sandwich, between two values); for "greater than," you want the points *far* from zero (two pieces heading outward).

ABS If an absolute-value equation equals a negative number, it has no solution. For absolute-value inequalities, check $c < 0$ as a special case because distance is never negative.

Visual: close to zero vs. far from zero





Translating Words into Algebra

Half of algebra is turning English into symbols. Here's your dictionary.

- sum / increased by / more than $\to +$
- difference / less than / fewer $\to -$
- product / of / times / twice $\to \times$
- quotient / per / split $\to \div$
- is / was / equals / gives $\to =$
- a number / unknown $\to x$

WORDS "5 less than a number" is $x - 5$, *not* $5 - x$. The order matters because "less than" reverses the order in the expression.

3 Functions, Lines & Slope

A function is one of the biggest ideas in all of math: a reliable machine that turns each input into exactly one output. Lines are the simplest functions, and slope is the number that describes how steep they are.

What a Function Is

Definition

A **function** gives exactly one output y for each input x. We write $y = f(x)$. **Vertical Line Test:** a graph is a function if no vertical line ever hits it twice.

Tutor's Note

Think of a vending machine: press B4 and you always get the same snack. That "same input, same single output" rule is what makes something a function. The notation $f(x)$ is read "f of x" and just means "the output when the input is x." It is *not* multiplication — it's a name tag for the machine's result.

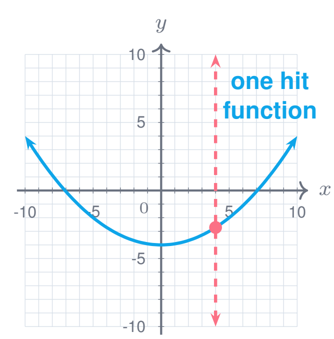
Visual: vertical line test

How to read it

A graph is a function when each input x has only one output y.

The dashed vertical line touches this curve once, so it passes the test.

Domain — allowed inputs.
Range — possible outputs.
Evaluate: $f(3) = 2(3) + 1 = 7$.



Slope — Steepness in a Number

Slope between two points

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} \quad (x_2 \neq x_1)$$

Tutor's Note

Slope answers "for every step I take to the right, how far do I go up or down?" That's the *rise over run*. A big slope is a steep hill; a small slope is a gentle ramp. The sign tells direction: positive climbs as you read left to right, negative descends.

SLOPE

Subtract the x's and the y's in the *same order*. Mixing the order flips the sign and changes the slope.

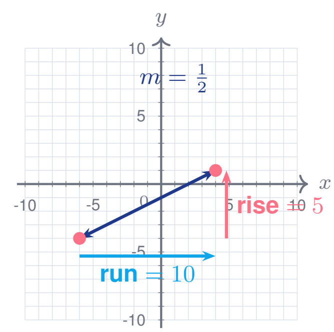
Visual: rise over run

Slope is a rate

Count the horizontal change first, then the vertical change.

Here, run $= 10$ and rise $= 5$, so $m = \frac{5}{10} = \frac{1}{2}$.

Positive slope rises left to right. Negative slope falls left to right.



- $m > 0$: line rises (↗)
- $m < 0$: line falls (↘)
- $m = 0$: flat, horizontal $y = b$
- m undefined: vertical $x = a$

Forms of a Linear Equation

The same line can be written several ways. Each form is handy for a different job.

Slope-Intercept
$y = mx + b$
slope m, y-intercept b

Standard
$Ax + By = C$
(A and B not both 0)

Point-Slope
$y - y_1 = m(x - x_1)$

Intercepts
x-int: set $y = 0$
y-int: set $x = 0$

Tutor's Note

Use **slope-intercept** when you know the slope and where the line crosses the y-axis — you can graph it instantly. Use **point-slope** when you know the slope and *any* one point; it's the fastest way to write a line's equation from scratch. **Standard form** is tidy for finding both intercepts. They all describe the same line — just dressed for different occasions.

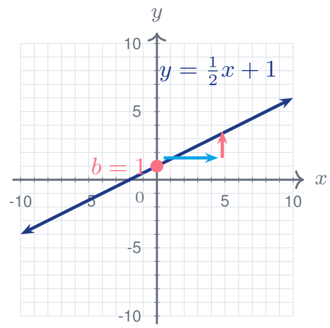
Visual: slope-intercept form

Use slope-intercept form

The line crosses the y-axis at $b = 1$.

The slope is $\frac{1}{2}$: move right 2, then up 1; or right 4, then up 2.

The arrows show the line continues forever in both directions.



Parallel & Perpendicular Lines

It's all in the slopes

Parallel: $m_1 = m_2$ **Perpendicular:** $m_1 \cdot m_2 = -1$ (negative reciprocals, for nonvertical lines)

Tutor's Note

Parallel lines never meet, so they must rise at the exact same rate — equal slopes. Perpendicular lines cross at a right angle, and it turns out their slopes are "negative reciprocals": flip the fraction and change the sign. So a slope of 2 is perpendicular to $-\frac{1}{2}$. A quick check: multiply them and you should get -1.

LINES

A vertical line and a horizontal line are perpendicular, but the vertical line has undefined slope, so the product rule does not apply to that pair.

Distance & Midpoint

Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Tutor's Note

The distance formula is secretly the Pythagorean theorem in disguise: the horizontal gap and vertical gap are the two legs of a right triangle, and the distance is the hypotenuse. The midpoint is even friendlier — it's just the average of the two x's and the average of the two y's, landing you exactly halfway.

Direct & Inverse Variation

Direct: $y = kx$

Inverse: $y = \dfrac{k}{x} \quad (x \neq 0,\ k \neq 0)$, or $xy = k$

Tutor's Note

Direct variation is a straight line through the origin — double the input, double the output when k is positive (think hours worked and pay earned). If k is negative, the line still passes through the origin but goes downward. **Inverse** variation is a constant-product trade-off — with positive quantities, more speed means less travel time. The constant k stays fixed and ties the two quantities together.

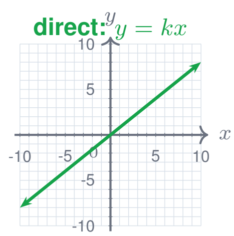
Visual: direct vs. inverse variation

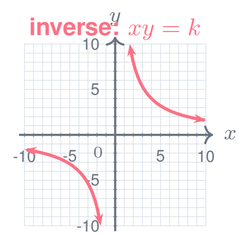
Compare the shapes

Direct variation is a line through the origin: $y = kx$.

Inverse variation bends in two branches because $xy = k$ stays fixed.

Both graphs continue beyond the viewing window.





4 Exponents & Scientific Notation

An exponent is just a shorthand for repeated multiplication. Once you see that, every exponent rule below stops being something to memorize and becomes something you can figure out.

The Laws of Exponents

These rules apply whenever every expression involved is defined. In this list, m and n are integers unless a fractional exponent is shown.

Product
$a^m \cdot a^n = a^{m+n}$

Power of a Power
$(a^m)^n = a^{mn}$

Quotient
$\dfrac{a^m}{a^n} = a^{m-n} \quad (a \neq 0)$

Power of a Product
$(ab)^n = a^n b^n$

Power of a Quotient
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad (b \neq 0)$

Negative Exponent
$a^{-n} = \frac{1}{a^n} \quad (a \neq 0)$

Zero Exponent
$a^0 = 1 \quad (a \neq 0)$

Fractional Exponent
$a^{m/n} = (\sqrt[n]{a})^m \quad (n \in \mathbb{Z}^+)$

Real-number note: with even roots, the base under the root must be nonnegative. For example, $\sqrt{x^2} = |x|$, not always x. With odd roots, negative radicands are allowed.

Tutor's Note

Why these all make sense $a^3 \cdot a^2$ means $(a \cdot a \cdot a)(a \cdot a)$ — five a's, so you *add* the exponents. Dividing cancels factors, so you *subtract*. A power of a power stacks groups, so you *multiply*. The strange ones follow the same logic: $a^0 = 1$ because $\frac{a^3}{a^3} = a^{3-3} = a^0$ and anything over itself is 1. A *negative* exponent just means "flip it to the other side of the fraction." Memorize the pattern once and the rest is bookkeeping.

Example

$\frac{2x^3 \cdot 3x^4}{x^2} = \frac{6x^7}{x^2} = 6x^5$. (Multiply coefficients, add then subtract exponents.)

Scientific Notation

A compact way to write very big or very small numbers without drowning in zeros.

Standard form

$$a \times 10^n, \qquad 1 \leq |a| < 10, \quad n \in \mathbb{Z}$$

Big numbers: $n > 0$. Numbers with $0 < |x| < 1$: $n < 0$. Zero is written as 0.

Tutor's Note

The power of 10 just counts how many places the decimal point moves. Positive powers push it right (making the number bigger); negative powers pull it left (smaller). To multiply two of these, multiply the front numbers and *add* the exponents; to divide, divide the fronts and *subtract* — exactly the exponent laws from above.

Example

$3{,}400 = 3.4 \times 10^3 \qquad 0.0052 = 5.2 \times 10^{-3}$.
$(2 \times 10^5)(3 \times 10^2) = 6 \times 10^7$.

Tip: The front number always has exactly one digit before the decimal point — that's what makes it "standard."

5 Polynomials & Factoring

A polynomial is just a sum of terms like $3x^2$, $-5x$, and 7. Multiplying them out and then factoring them back apart are two sides of the same coin — and factoring is the key that unlocks quadratics in the next section.

♦ Polynomial Vocabulary

Degree — the highest exponent in a nonzero polynomial.
Leading coefficient — the number on the highest-degree term.

Named by number of terms

- Monomial — 1 term
- Binomial — 2 terms
- Trinomial — 3 terms

Tutor's Note

"Like terms" are terms with the exact same variable part — $3x^2$ and $5x^2$ are like terms, but $3x^2$ and $3x$ are not. You can only combine like terms, the same way you can add apples to apples but not apples to oranges.

♦ Adding, Subtracting & Multiplying

Add / Subtract

Combine like terms.

$$(3x^2 + 2x) - (x^2 - 5x) = 2x^2 + 7x$$

Multiply — FOIL for two binomials

$$(a + b)(c + d) = ac + ad + bc + bd$$

First, **O**uter, **I**nner, **L**ast.

Tutor's Note

When subtracting, distribute the minus sign to *every* term in the second parentheses — that sign flip is the #1 source of errors here. For multiplying, FOIL is really just the distributive property done twice: every term in the first group shakes hands with every term in the second.

SIGNS

$-(x^2 - 5x)$ becomes $-x^2 + 5x$. A minus sign outside parentheses distributes to every term inside.

Special Products (memorize these — they save time)

Square of a Sum
$(a+b)^2 = a^2 + 2ab + b^2$

Difference of Squares
$(a+b)(a-b) = a^2 - b^2$

Square of a Difference
$(a-b)^2 = a^2 - 2ab + b^2$

Sum/Diff of Cubes
$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

Tutor's Note

These are just FOIL results worth remembering. Notice the "difference of squares" magic: the middle terms cancel, leaving only $a^2 - b^2$. And watch the common slip — $(a+b)^2$ is *not* $a^2 + b^2$; you must include that $2ab$ middle term.

Factoring — a Reliable Game Plan

Factoring reverses multiplication: it rewrites a polynomial as a product. Always work through these steps in order.

The factoring checklist

1. **GCF first.** Pull out the greatest common factor from every term.
2. **Count the terms:**
   - 2 terms $\rightarrow$ try difference of squares or sum/difference of cubes
   - 3 terms $\rightarrow$ trinomial factoring
   - 4 terms $\rightarrow$ factor by grouping
3. **Check** by multiplying your answer back out.

Tutor's Note

Factoring feels like a puzzle at first, but the checklist removes the guesswork. Step one is non-negotiable: always look for a common factor before anything else — it makes every later step easier. Then let the number of terms point you to the right tool.

Factoring Trinomials

When $a = 1$: find two numbers that *multiply* to c and *add* to b.

$$x^2 + bx + c = (x+p)(x+q), \qquad p+q=b,\ pq=c$$

When $a \neq 1$: use the *ac-method* — split the middle term, then group.

Example

$x^2 + 7x + 12$: we need two numbers that multiply to 12 and add to 7 — that's 3 and 4. So $x^2 + 7x + 12 = (x+3)(x+4)$.

Tutor's Note

This "multiply to c, add to b" trick works because when you FOIL $(x+p)(x+q)$ you get $x^2+(p+q)x+pq$. So the outside number c is the product and the middle number b is the sum. List the factor pairs of c and find the pair that adds up right.

6 Quadratic Functions & Equations

A quadratic has an x^2 in it, and its graph is a graceful U-shaped curve called a parabola. This is the showpiece of Algebra 1 — and once you have factoring and the quadratic formula, you can solve any of them.

Standard & Vertex Forms

Standard Form
$y = ax^2 + bx + c \quad (a \neq 0)$

Vertex Form
$y = a(x-h)^2 + k$
vertex at (h, k)

Tutor's Note

Both forms describe the same parabola. **Standard form** shows the y-intercept at a glance (it's c). **Vertex form** hands you the most important point — the very top or bottom of the U — directly as (h, k). The number a controls the shape: large a makes a narrow U, small a a wide one, and a negative a flips it upside down.

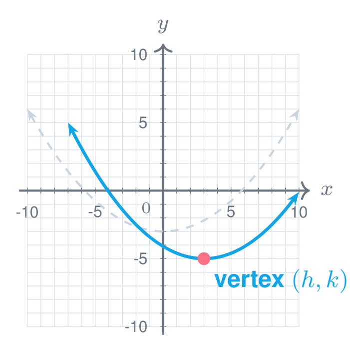
Visual: vertex form shifts the parabola

Vertex form

$y = a(x-h)^2 + k$ puts the vertex at (h, k).

The highlighted parabola has vertex $(3, -5)$, so the graph shifts right and down.

The arrows show each arm keeps rising outside the window.



Reading the Graph (the Parabola)

Key features

- Opens **up** if $a > 0$, **down** if $a < 0$.
- Axis of symmetry: $x = -\dfrac{b}{2a}$
- Vertex: plug that x back in to get y.
- y-intercept: $(0,\ c)$

Tutor's Note

Every parabola is perfectly symmetric, like a mirror folded down the middle. That mirror line is the axis of symmetry, found by $x = -\frac{b}{2a}$. The vertex sits right on that line — it's the lowest point if the U opens up, or the highest if it opens down. That makes the vertex the key to any "maximum height" or "minimum cost" problem.

Visual: key parabola features

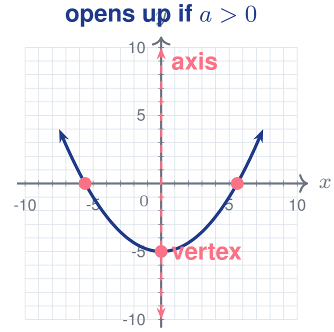
Parabola checklist

Find the axis first: $x = -\frac{b}{2a}$.

The vertex sits on the axis. The x-intercepts are where the curve crosses the x-axis.

Since $a > 0$, the parabola opens upward.

opens up if $a > 0$



Solving Quadratic Equations

The Quadratic Formula — it always works

$$ax^2 + bx + c = 0 \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (a \neq 0)$$

Tutor's Note

You have a toolbox of methods — reach for the easiest one that fits:

- **Factoring** (fastest *when* it factors): if $pq = 0$, then $p = 0$ or $q = 0$.
- **Square roots:** if $x^2 = k$, then $x = \pm\sqrt{k}$ for $k \geq 0$.
- **Completing the square:** rewrites it in vertex form.

Completing the square

$$x^2 + bx = \left(x + \tfrac{b}{2}\right)^2 - \left(\tfrac{b}{2}\right)^2$$

Tutor's Note

The "zero-product" idea is sneaky-powerful: if two things multiply to zero, at least one of them must *be* zero. That's why factoring solves equations — set each factor to zero. When factoring fails, the quadratic formula never does; it's the universal key that works on every quadratic, no matter how ugly.

The Discriminant — a Sneak Peek at the Answers

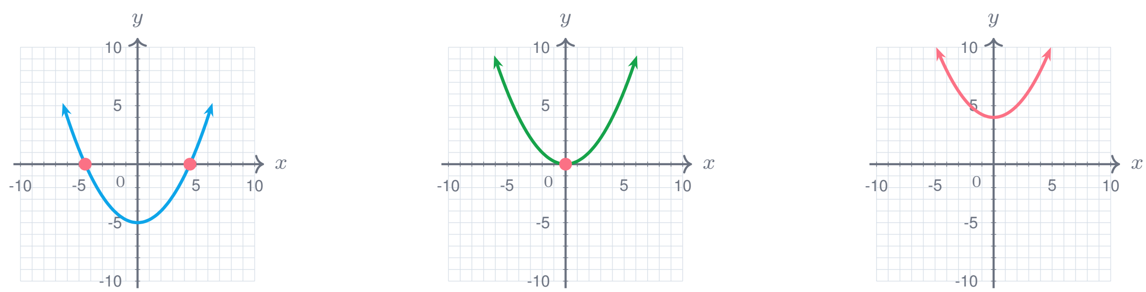
$D = b^2 - 4ac$ counts the real solutions

- $D > 0$: two different real solutions
- $D = 0$: one repeated real solution
- $D < 0$: no real solutions (the parabola misses the x-axis)

Tutor's Note

The discriminant is the part under the square root. Before solving, it tells you what to expect: a positive value means the parabola crosses the x-axis twice, zero means it just touches once, and a negative means it floats entirely above or below — no real crossing, because you can't take the square root of a negative in real numbers.

Visual: discriminant and x-intercepts



$D > 0$: **two x-intercepts** $D = 0$: **one touch** $D < 0$: **no real intercepts**

Example

$x^2 - 5x + 6 = 0$. It factors: $(x-2)(x-3) = 0$, so $x = 2$ or $x = 3$. Check: $D = (-5)^2 - 4(1)(6) = 25 - 24 = 1 > 0$, confirming two real answers.

7 Radicals & Rational Expressions

A radical (square root) undoes a square, and a rational expression is just a fraction with variables. Both follow rules you already know from numbers — we're simply letting letters join in.

Radical Rules

Product
$\sqrt{ab} = \sqrt{a}\sqrt{b} \quad (a, b \geq 0)$

Radical = Exponent
$\sqrt[n]{a} = a^{1/n} \quad (n \in \mathbb{Z}^+)$

Quotient
$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \quad (a \geq 0,\ b > 0)$

Power
$a^{m/n} = (\sqrt[n]{a})^m$

Real-number note: even roots need a nonnegative radicand. Also, $\sqrt{x^2} = |x|$ because the principal square root is never negative. Odd roots can have negative radicands.

Tutor's Note

The most freeing idea here: a root is just a *fractional exponent*. A square root is the $\frac{1}{2}$ power, a cube root is the $\frac{1}{3}$ power. Once you see that, all the exponent laws from Section 4 apply to radicals too — you don't need a separate rule book. The product rule also lets you "break apart" a root to simplify it.

Simplifying & Rationalizing

Simplify a radical

Pull out perfect-square factors: $\sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}$.

Rationalize the denominator

Multiply by a clever form of 1 to clear the root from the bottom:

$$\frac{1}{\sqrt{a}} \cdot \frac{\sqrt{a}}{\sqrt{a}} = \frac{\sqrt{a}}{a} \quad (a > 0) \qquad \frac{1}{a + \sqrt{b}} \cdot \frac{a - \sqrt{b}}{a - \sqrt{b}} = \frac{a - \sqrt{b}}{a^2 - b} \quad (b \geq 0,\ a^2 \neq b)$$

The flipped-sign partner $a - \sqrt{b}$ is called the **conjugate**.

Tutor's Note

To simplify, hunt for the biggest perfect square hiding inside (4, 9, 16, 25, . . .) and take its root out front. "Rationalizing" just means cleaning roots out of the denominator, which is considered tidy form. Multiplying by the conjugate works because of difference of squares — it turns $\sqrt{b}$ into plain b and the messy root vanishes.

Solving Radical Equations

Isolate, then undo the root

Get the radical alone, raise both sides to the matching power, then **always check** your answers in the original equation.

$$\sqrt{x+3} = 5 \Rightarrow x+3 = 25 \Rightarrow x = 22$$

Tutor's Note

Squaring both sides is the move that frees x from under the root. But squaring can secretly introduce a fake answer (an "extraneous" solution), so checking isn't optional here — it's part of the method. Plug your answer back in; if it doesn't actually work, toss it.

CHECK

After squaring both sides, check in the *original* equation, not the squared version. The squared equation can accidentally accept fake answers.

Rational Expressions

Simplify by canceling common factors:

$$\frac{x^2-9}{x+3} = \frac{(x-3)(x+3)}{x+3} = x-3, \qquad x \neq -3$$

Multiply / **Divide** like any fractions:

$$\frac{a}{b}\cdot\frac{c}{d} = \frac{ac}{bd}, \quad \frac{a}{b} \div \frac{c}{d} = \frac{a}{b}\cdot\frac{d}{c} \quad (b,d,c \neq 0)$$

Restriction: a denominator can never equal 0. Exclude any x that would make the bottom zero — those values are off-limits.

Tutor's Note

To simplify a rational expression, factor the top and bottom first, then cancel matching factors — exactly like reducing $\frac{6}{9}$ to $\frac{2}{3}$. Just remember the one rule the calculator can't bend: dividing by zero is undefined, so always note which x-values are forbidden before you simplify.

8 Systems of Equations & Inequalities

A system is two (or more) equations that must be true at the same time. The solution is the point where they agree — where their graphs cross.

Three Ways to Solve

Graphing

Plot both lines; the solution is the point where they intersect.

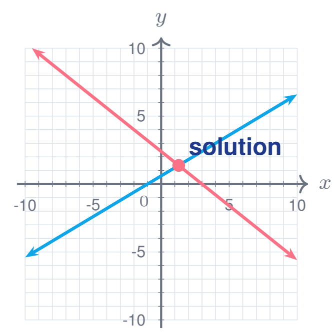
Visual: solution is the intersection

Graphing a system

Draw both equations on the same plane.

The solution is the point that lies on both lines at the same time.

If the lines do not meet, there is no solution. If they are the same line, there are infinitely many.



Substitution

Solve one equation for a variable, then plug it into the other.

$$\begin{cases} y = 2x \\ x + y = 6 \end{cases} \Rightarrow x + 2x = 6 \Rightarrow x = 2,\ y = 4$$

Elimination

Add or subtract the equations to cancel a variable.

$$\begin{cases} x + y = 7 \\ x - y = 1 \end{cases} \Rightarrow 2x = 8 \Rightarrow x = 4,\ y = 3$$

Tutor's Note

Which method should I pick? **Graphing** is great for seeing what's going on, but it's only as exact as your drawing. **Substitution** shines when one variable is already alone (like $y = 2x$). **Elimination** is fastest when the equations line up so a variable cancels when you add them. All three reach the same answer — choose whichever fits the problem in front of you.

How Many Solutions?

One solution
Lines cross once.
Different slopes.

Infinitely many
Same line.
Same slope and same intercept.

No solution
Parallel lines.
Same slope, different intercept.

Tutor's Note

Think geometrically. Two lines that lean differently must cross at exactly one point — one solution. Two parallel lines never meet — no solution. And if both equations secretly describe the *same* line, every point works — infinitely many. The slopes and intercepts tell you which case you're in before you even solve.

Systems of Inequalities

Graph, then shade

- Solid line for $\leq$, $\geq$; dashed line for $<$, $>$.
- Shade the side of each line that makes its inequality true.
- The **solution** is the region where the shadings overlap.

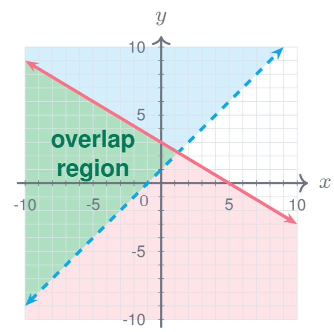
Visual: overlapping solution region

Shade and overlap

Use a dashed boundary for $<$ or $>$ and a solid boundary for $\leq$ or $\geq$.

The answer is the region where both shaded half-planes overlap.

Test a point, often $(0, 0)$, to decide which side to shade.



Tutor's Note

With inequalities, the answer is a whole *region*, not one point. A solid boundary counts; a dashed boundary does not. The overlapping shaded zone is the solution. **Tip:** To choose a side, test $(0, 0)$ if it is not on the boundary.

9 Sequences & Statistics

We finish with two practical topics: patterns of numbers (sequences) and ways to summarize data (statistics). Both show up constantly in real life and on tests.

Arithmetic Sequences

Add the same amount each time (d)

$$a_n = a_1 + (n-1)d$$

a_1 is the first term and d is the common difference.

Tutor's Note

An arithmetic sequence grows by *adding* a fixed step every time — like stairs of equal height. The formula just says: start at a_1, then take $(n-1)$ steps of size d to reach the nth term. The $(n-1)$ catches a lot of students — the first term takes *zero* steps, so we subtract one.

Example

$3, 7, 11, 15, \ldots$ here $a_1 = 3$ and $d = 4$, so $a_n = 3 + (n-1)4 = 4n - 1$.

Geometric Sequences

Multiply by the same amount each time (r)

$$a_n = a_1 \cdot r^{n-1}$$

r is the common ratio.

Tutor's Note

A geometric sequence grows by *multiplying* by a fixed ratio — this is how money compounds and how populations grow. Same structure as before, but repeated multiplication replaces repeated addition, so r is raised to the $(n-1)$ power.

Example

$2, 6, 18, 54, \ldots$ here $a_1 = 2$ and $r = 3$, so $a_n = 2 \cdot 3^{n-1}$.

Measures of Center & Spread

Mean (average)

$\bar{x} = \dfrac{\sum x}{n}$

Median

the middle of ordered data; if there are two middles, average them

Mode

the most frequent value

Range

$\max - \min$

Tutor's Note

Mean is the everyday "average" — add up everything and share it equally. **Median** is the middle value when data is lined up in order, and it's the better choice when a few extreme values would distort the mean. **Mode** is simply the most common value, and **range** measures how spread out the data is.

DATA

Always sort the data from low to high *before* finding the median. If there are two middle values, average those two.

Probability

How likely is it?

$$P(\text{event}) = \frac{\text{favorable outcomes}}{\text{total outcomes}}$$

$0 \le P \le 1.$ $P(\text{not } A) = 1 - P(A).$ Assumes equally likely outcomes.

Independent events (one doesn't affect the other)

$P(A \text{ and } B) = P(A) \cdot P(B)$ (independent)

Either event

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

If events are mutually exclusive, $P(A \text{ and } B) = 0$.

Tutor's Note

Probability is always a number between 0 (impossible) and 1 (certain), often written as a fraction or percent. For *independent* events, "and" means multiply the chances. For "or," add the chances but subtract any overlap so it is not counted twice. If the events are *mutually exclusive*, there is no overlap, so "or" simply means add them. A handy shortcut: the chance something *doesn't* happen is 1 minus the chance it does.

You made it through every Algebra 1 formula and the reasoning behind each one!

Review a little each day, work a few problems, and these ideas become second nature.

Keep going — you're closer than you think.

FINAL

Before a test, do one last scan for restrictions: denominators cannot be 0, square roots with even indexes need nonnegative radicands, and answers from squared or rational equations should be checked in the original problem.

Algebra Symbols Cheat Sheet

Symbol • Meaning • Example

Comparisons & Solutions

Symbol	Meaning	Example
$=$	equals	$x = 5$
$\neq$	not equal	$x \neq 0$
$<$	less than	$x < 3$
$>$	greater than	$x > 3$
$\leq$	less than or equal to	$x \leq 7$
$\geq$	greater than or equal to	$x \geq -2$
$\approx$	approximately equal	$\pi \approx 3.14$
$\pm$	plus or minus	$x = \pm 4$
$\Rightarrow$	implies / leads to	$2x = 8 \Rightarrow x = 4$
$\Leftrightarrow$	equivalent statements	$x + 1 = 5 \Leftrightarrow x = 4$
$\varnothing$	no solution / empty set	$x = x + 1 \Rightarrow \varnothing$
∞	continues forever	many solutions

Sets, Intervals & Numbers

Symbol	Meaning	Example
$\{\ \}$	set / solution list	$\{1, 2, 3\}$
$\in$	is an element of	$3 \in \{1, 3, 5\}$
$\notin$	is not an element of	$4 \notin \{1, 3, 5\}$
$\mathbb{R}$	real numbers	$x \in \mathbb{R}$
$\mathbb{Z}$	integers	$\ldots, -1, 0, 1, \ldots$
$\mathbb{Q}$	rational numbers	$\frac{3}{4} \in \mathbb{Q}$
(a, b)	open interval	$2 < x < 5$
$[a, b]$	closed interval	$2 \leq x \leq 5$
$[a, b)$	half-open interval	$2 \leq x < 5$
$\cup$	union / OR	$x < 1 \cup x > 4$
$\cap$	intersection / AND	$x > 1 \cap x < 4$
$\mid$	"such that"	$\{x \mid x > 0\}$

Operations & Powers

Symbol	Meaning	Example
$+$	add / positive	$3 + 5 = 8$
$-$	subtract / negative	$7 - 10 = -3$
$\cdot, \times$	multiply	$4 \cdot x = 4x$
$\div, /$	divide	$12 \div 3 = 4$
$\frac{a}{b}$	fraction / quotient	$\frac{x}{2} = 5$
$\%$	percent, per hundred	$20\% = 0.20$
x^2	square	$5^2 = 25$
x^n	exponent / power	$2^3 = 8$
$\sqrt{x}$	square root	$\sqrt{49} = 7$
$\sqrt[n]{x}$	nth root	$\sqrt[3]{8} = 2$
$\lvert x \rvert$	absolute value	$\lvert -6 \rvert = 6$
$()\ []\ \{\}$	grouping symbols	$2(x + 3)$

Functions, Graphs & Data

Symbol	Meaning	Example
(x, y)	ordered pair / point	$(2, 5)$
$f(x)$	output of a function	$f(3) = 7$
m	slope of a line	$y = mx + b$
b	y-intercept	$y = 2x + 5$
Δ	change in	$m = \frac{\Delta y}{\Delta x}$
$\sum$	add them all	$\sum x = x_1 + x_2 + \cdots$
$\bar{x}$	mean / average	$\bar{x} = \frac{\sum x}{n}$
a_n	nth term	$a_n = a_1 + (n - 1)d$
D	discriminant	$D = b^2 - 4ac$
$P(A)$	probability of event A	$P(A) = \frac{3}{10}$
$A \cap B$	A and B	overlap counted once
$A \cup B$	A or B	either event happens

Algebra Vocabulary Bank

Vocabulary is part of the math. When the word is clear, the formula is easier to choose and the graph is easier to read.

Foundations & Expressions

Variable — a letter that represents a number.
Constant — a number that does not change.
Coefficient — the number multiplying a variable.
Term — one piece of an expression, separated by $+$ or $-$.
Expression — numbers, variables, and operations with no equals sign.
Equation — a statement that two expressions are equal.
Factor — a number or expression being multiplied.
Multiple — a result of multiplying by an integer.
Like terms — terms with the same variable part.
Equivalent expressions — expressions with the same value for every allowed input.
Distributive property — multiplying a factor across a sum or difference.
Simplify — rewrite in a cleaner equivalent form.
Evaluate — find the value when variables are known.
Substitute — replace a variable with a number or expression.
Order of operations — the agreed order for simplifying expressions.

Functions & Lines

Relation — a set of input-output pairs.
Function — a relation with exactly one output for each input.
Function notation — writing outputs as $f(x)$, read as "f of x".
Input — the x-value.
Output — the y-value or $f(x)$.
Domain — all allowed input values.
Range — all possible output values.
Discrete graph — separate points, not a connected curve.
Continuous graph — a connected graph over an interval.
Vertical line test — a graph test for whether a relation is a function.
Slope — rate of change; rise divided by run.
Rate of change — how fast one quantity changes compared with another.
x-intercept — where a graph crosses the x-axis.
y-intercept — where a graph crosses the y-axis.
Slope-intercept form — $y = mx + b$.
Point-slope form — $y - y_1 = m(x - x_1)$.
Standard form — $Ax + By = C$.
Parallel lines — lines with the same slope.
Perpendicular lines — lines that meet at a right angle.

Equations & Inequalities

Solution — a value that makes a statement true.
Solution set — the complete collection of values that work.
Solve — find all solutions.
Inverse operations — operations that undo each other.
Isolate — get the variable alone.
Literal equation — an equation with more than one variable.
Identity — an equation true for every allowed value.
Contradiction — an equation that is never true.
No solution — when no allowed value makes the statement true.
All real numbers — every real number is a solution.
Inequality — a comparison using $<, >, \leq$, or $\geq$.
Compound inequality — two inequalities joined by AND or OR.
Open dot — endpoint not included.
Closed dot — endpoint included.
Interval notation — a compact way to write a set of numbers.

Systems & Graphing

System — two or more equations or inequalities considered together.
Solution to a system — a point or region that makes every statement true.
Substitution — solve by replacing one expression with an equal one.
Elimination — solve by adding equations to cancel a variable.
Intersection — where graphs meet.
Boundary line — the edge line of a linear inequality.
Half-plane — one side of a boundary line.
Solution region — the shaded region that satisfies an inequality.
Linear model — a line used to represent a real situation.
Direct variation — a relationship of the form $y = kx$.
Inverse variation — a relationship of the form $xy = k$.
Consistent system — a system with at least one solution.
Inconsistent system — a system with no solution.
Dependent system — equations that describe the same line or relationship.

WORDS

Translate the command before you calculate: simplify cleans an expression, solve finds values, evaluate plugs in values, and graph shows the solution visually.

Algebra Vocabulary Bank

Powers • Polynomials • Quadratics • Data

Exponents, Radicals & Rational Expressions

Base — the number or expression being raised to a power.
Exponent — the power that tells how many factors of the base are used.
Power — an expression like x^n.
Square root — a number that squares to the radicand.
Principal square root — the nonnegative square root.
Radical — a root symbol such as $\sqrt{\ }$.
Radicand — the expression under a radical.
Index — the root number in $\sqrt[n]{x}$.
Rationalize — remove a radical from a denominator.
Conjugate — a binomial with the middle sign changed.
Rational expression — a fraction with variables.
Restriction — a value that is not allowed, often because it makes a denominator 0.
Extraneous solution — an answer created by algebra steps that fails the original problem.

Quadratics

Quadratic equation — an equation with highest power 2.
Quadratic function — a function that can be written $f(x) = ax^2 + bx + c$.
Standard form — $ax^2 + bx + c$ or $ax^2 + bx + c = 0$.
Vertex form — $a(x-h)^2 + k$, which shows the vertex.
Parabola — the U-shaped graph of a quadratic function.
Vertex — the turning point of a parabola.
Axis of symmetry — the vertical line through the vertex.
Minimum — the lowest value of a graph.
Maximum — the highest value of a graph.
Root — a solution of an equation.
Zero — an input that makes the output 0.
x-intercept — where the parabola crosses the x-axis.
Discriminant — $b^2 - 4ac$, which predicts the number of real solutions.
Completing the square — rewriting a quadratic to make a perfect square.
Quadratic formula — $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Polynomials & Factoring

Polynomial — a sum of terms with whole-number exponents.
Monomial — a polynomial with one term.
Binomial — a polynomial with two terms.
Trinomial — a polynomial with three terms.
Degree — the highest exponent in a polynomial.
Leading coefficient — the coefficient of the highest-degree term.
Standard form — terms written from highest degree to lowest degree.
FOIL — first, outer, inner, last for multiplying binomials.
GCF — greatest common factor.
Factored form — a product of factors.
Factor completely — rewrite as a product with no more common factors.
Prime polynomial — a polynomial that cannot be factored further in the given number system.
Difference of squares — $a^2 - b^2 = (a+b)(a-b)$.
Perfect-square trinomial — a trinomial that factors as a binomial squared.

Sequences, Data & Probability

Sequence — an ordered list of numbers.
Arithmetic sequence — a sequence that adds the same amount each time.
Common difference — the repeated amount added in an arithmetic sequence.
Geometric sequence — a sequence that multiplies by the same amount each time.
Common ratio — the repeated multiplier in a geometric sequence.
Mean — the average; add values and divide by how many there are.
Median — the middle value after sorting.
Mode — the most frequent value.
Range — maximum minus minimum.
Outlier — a data value far from most of the others.
Scatter plot — a graph of paired data points.
Line of best fit — a line that models the trend in a scatter plot.
Correlation — how strongly two quantities are related.
Probability — a number from 0 to 1 that describes chance.
Independent events — events where one result does not affect the other.
Mutually exclusive — events that cannot happen at the same time.

VOCAB If a word appears in a question, pause and translate it first. In Algebra 1, vocabulary often tells you which operation, graph, or formula to use.

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REFERENCE
Formula sheet
All key formulas in one place, connected to practice tools.

Best loop: review the formula here, scan the hub, practice one weak skill, then return to mixed review. EffortlessMath.com/algebra-1